**Differential Equation Method for determining the contribution of intumescent coatings to the fire resistance of structural steelwork**

**Thermal behaviour**

1. Determination of the relationship between the time and the average steel temperature.

   Determine for each of the test specimens required for the assessment of the thermal behaviour, the relationship between the time and the average steel temperature.

   The average steel temperature is the mean of the measured temperatures of the thermocouples on the test specimen in $^0$C for time steps t>0.

2. Determination of $K_{eff}$ for intumescent coatings.

$$K = \frac{\lambda}{d} \qquad [1]$$

where:

K heat transfer coefficient of intumescent coating [W/m$^2$ $^0$C]

$\lambda$ thermal conductivity of intumescent coating [W/m $^0$C]

d thickness of intumescent coating [m]

a) Determine for each of the temperature-time relationships as determined in Section 1. The value of factor K as a function of the steel temperature values specified hereafter $(\theta_{a;}\ \text{spe})$, using the temperature development of the gas temperatures in the furnace $(\theta_t)$ based on the mean value of the thermocouples for measuring the gas temperatures in the furnace and using the following equation for the calculation of the temperature increase $\Delta\theta_a$ of the steel section for a time interval $\Delta_t$ :

$$\Delta\theta_{a,t} = \frac{K}{C_a\,\rho_a} A/V\left(\theta_t - \theta_{a,t}\right)\Delta t \qquad [2]$$

where:

K = heat transfer coefficient

$C_a$ = specific heat of steel [J/kg$^0$C]

$\rho_a$ = density of steel [kg/m$^3$]

A/V = section factor of the protected steel section [$m^{-1}$]

$\theta_t$ = average temperature of the furnace at time t [$^0C$]

$\theta_{a,t}$ = average temperature of the steel at time t [$^0C$]

$\Delta t$ = the time interval [sec]

in such a way that for each value of $\theta_{a,spe}$ the following requirement has been satisfied.

$$t_{\theta_{a,calc}} \leq t_{\theta_{a,exp}} \pm 1 \qquad [3]$$

where:

$t_{\theta_{a,calc}}$ is the time, in minutes, for which, using a specific value of K, a steel temperature is calculated which equals the specified value of $\theta_{a,spe}$.

$t_{\theta_{a,exp}}$ is the time, in minutes, for which, on the basis of the experimental test, an average steel temperature is achieved which is equal to the specified value of $\theta_{a,spe}$

Take for $\theta_{a,spe}$ multiple values of $50^0C$ with a starting point of $400^0C$ and an end value of $750^0C$.

The principle of this analysis is indicated in Figure 1.

b) Present the values of $K_d$ determined in the analysis above, multiplied with the related value of the intumescent coating thickness $d_i$, as a function of the steel temperature $\theta_a$, the protection thickness (= dry film thickness) $d_i$, and the section factor $A_i / V_i$.

c) Determine, using the method of least squares, the following equation:

$$K_{d,i} = K_{d,eff} = \frac{1}{d} \{C_o + C_1\ \theta_a + C_2\ d_i + C_3\ A_i / V_i\} \qquad [4]$$

where:

$C_o$, $C_1$, $C_2$ and $C_3$ are constants.

In the method of least squares a line fits that minimises the sum of the squared residuals, ie. a set of fitted values is found that minimises the sum:

$$\sum_{i=1}^{n} r_i^2$$

The principle of the determination for one specific value of $A_l/V_i$ is presented in Figure 2.

d) Calculate the values of time $t_{\theta_{a,calc}}$ and compare with values of time $t_{\theta_{a,exp}}$, using the specific values of $K_{d,eff}$

The calculated times are indicated as $t_{\theta_{a,calc}}$

The measured times in the fire test are indicated as $t_{\theta_{a,exp}}$

Determine for each of the steel temperatures $\theta_{a,spe}$ for each of the test specimens the ratio $t_{\theta_{a,calc}} / t_{\theta_{a,exp}}$

Furthermore, determine the arithmetic mean value of all ratios.

3. Processing of results.

a) The equation [4] as derived in Section 2c for determination of $K_{d,eff}$ may be used to calculate the temperature in steel sections, if the following criteria have been satisfied:

- all values of the ratio $t_{\theta_{a,calc}} / t_{\theta_{a,exp}}$ determined in accordance with section 2d, should be less than or equal to 1.3.
- The mean value of all ratios as calculated in Section 2d, shall be less than 1.0.

b) If the above mentioned criteria are not satisfied, the equation [4] for calculation of $K_{d,eff}$ has to be modified and the calculation steps in accordance with Section 2d, should be repeated.

## APPENDIX A

**Assessment of the contribution to the fire resistance of structural steelwork provided by intumescent coatings using the Differential Equation method**

The steel element is simplified as a heat sink and that it requires a certain amount of heat input as a function of its surface area in order for the steel temperature to increase as a function of time. The steel section has a heat content which is expressed in Joules as follows:

$$Q = \rho c v \Delta T \qquad [1]$$

where:

Q = heat content [J]

$\rho$ = density of steel [kg/m$^3$] (= 7850)

C = specific heat of steel [J/kg$^0$C] (= temperature dependent, approx 600 at 400$^0$C)

V = volume of the steel section per meter length [m$^3$]

$\Delta T$ = temperature difference [$^0$C]

The heat content of a 1m long steel section of type HE200A can be calculated as follows:

Q = 7850 . 600 . 4,534 . $10^{-3}$ . 1
= 21359 J per $^0$C

The above means that 21359J are needed to increase the temperature of a 1m long HE200A steel section by 1$^0$C.

If the HE200A section is provided with an intumescent coating then there will be a thermal resistance around the steel section which reduces the heat transfer to the steel section. The heat flowing from the furnace into the steel section can be calculated as follows:

$$q = \frac{(T_f - T_a)}{R} A \qquad [2]$$

where:

q = heat conduction rate [J/s = W]

$T_f$ = temperature of the furnace gases $^0$C

$T_a$ = steel temperature $^0$C

R = thermal resistance [m$^2$ $^0$C/W]

A = surface area of the steel section [$m^2$]

The amount of heat [J] that goes into the steel section per interval of time is a measure for the temperature increase of the steel section. It is evident that the greater the heat flow to the heat sink per time unit the quicker the 'heat sink' will rise in temperature. The steel temperature increase $\Delta T_a$ per time interval $\Delta t$ can now be calculated as follows:

Equation [2] can be written as:

$$\frac{q\ R}{A} = T_f - T_a \qquad [3]$$

Using Equation [1], Equation [3] can be written as:

$$\frac{p\ c\ V\ \Delta T\ R}{A} = \left(T_f - T_a\right)\Delta t \qquad [4]$$

The thermal resistance R is usually written as:

$$R = \frac{d}{\lambda} \qquad [5]$$

where:

d = thickness of the intumescent coating [m]

$\lambda$ = thermal conductivity [W/mK]

Intumescent coatings have non-constant behaviour with respect to reduction of heat transfer as they change in thickness (expanding intumescence) and they change with respect to heat transfer (bubbly crust), therefore the heat conductivity of intumescent coatings $\lambda$ is often written as:

$$\lambda = K\ d \qquad [6]$$

where:

K = thermal heat transfer coefficient

As $R = \frac{d}{\lambda}$ [5] and $\lambda = K\ d$ [6] the thermal resistance R can be written as:

$$R = \frac{d}{\lambda} = \frac{1}{K} \qquad [7]$$

Equation [4] can now be written as follows:

$$\Delta T_a = \frac{\left(T_f - T_a\right)}{\rho c V R} A \, \Delta t \qquad [8]$$

Using K for the heat transfer, Equation [8] goes over in:

$$\Delta T_a = \frac{K}{\rho c V} A (T_f - T_a) \Delta t \qquad [9]$$

as A [$m^2$] and V [$m^3$] are $\frac{A}{V}$ which equals Hp/A [$m^{-1}$] the section factor Hp/A can be introduced:

$$\Delta T_a = \frac{K}{\rho c} \text{ Hp/A } (T_f - T_a) \Delta t \qquad [10]$$

where:

| | | |
|---|---|---|
| $\Delta T_a$ | = | temperature rise of the steel section per unit of time [$^0$C] |
| Hp/A | = | section factor [$m^{-1}$] |
| $T_f$ | = | temperature of the furnace gases at time t [$^0$C] |
| $T_a$ | = | measured steel temperature during the fire test at time t [$^0$C] |
| $\Delta t$ | = | time interval (time step) [s] |

Equation [10] can be used to calculate the temperature rise of the steel section $\Delta T_a$ per time step $\Delta t$, provided that the heat transfer coefficient K is known.

However, K is not known and it is a great challenge to find a value for K, as K is the coefficient that characterises the performance of the intumescent coating. K is dependent on the thickness of the intumescent coating d, on the design steel temperature $T_{a\,spe}$ and on the section factor Hp/A.

The heat conductivity of the intumescent coating can be expressed as:

$$\lambda = C_o + C_1 \, T_{a\,spe} + C_2 \, d + C_3 \text{ Hp/A} \qquad [11]$$

where:

$T_{a\,spe}$ = design temperature for structural steelwork [$^0$C]

d = thickness of intumescent coating [m]

Hp/A = section factor [m$^{-1}$]

as $\lambda = K\ d$ [6] the Equation [11] can be written as:

$$K = \frac{1}{d}\left(C_0 + C_1 T_{a\,spe} + C_2 d + C_3 Hp/A\right) \qquad [12]$$

The temperature $T_{a\,spe}$ is the design temperature for the steel member and is dependent on the load on the steel structure; the higher the load the lower the design temperature. Heavily loaded steel structures, have a low design steel temperature and as lower design temperatures are achieved earlier than high design temperatures, heavily loaded structural steel sections need a thicker protective layer (more resistance against heat transfer) than structural elements with a low load if they are to satisfy a specific fire resistance time period, eg. 30 or 60 minutes.

Design steel temperatures are usually chosen in the range from 400$^0$C to 750$^0$C, considered for convenience, in 50$^0$C intervals. Hence $T_{a\,spe}$ = 400$^0$C, 450$^0$C, 500$^0$C…..750$^0$C.

For every specimen tested, the measured steel temperatures on the steel section are averaged in order to calculate the average steel temperature of the section as a function of the heating time. This is steel temperature $\theta_a$ [$^0$C] referred to in Equation [10]. When the steel temperature $\theta_a$ is available we can now calculate the time to achieve each of the design temperatures $T_{a\,spe}$, eg. it may take 21 minutes to achieve 400$^0$C, 26 minutes to achieve 450$^0$C and……68 minutes to achieve 750$^0$C.

These times to achieve a certain design temperature are calculated for every test specimen tested.

Using the differential Equation [10]

Now we have all of the input data available to start using Equation [10] in order to determine coefficient K. For every time step $\Delta t$, the steel temperature will rise by $\Delta T_a$. The actual steel temperature can now be calculated as function of time by summating all temperature rises $\Delta T_a$ above the initial steel temperature at commencement of the fire test, eg. 20$^0$C.

However, coefficient K in Equation [10] is not known and has to be determined by means of an iterative process in such a way that the calculated time to achieve a certain design temperature matches the actual measured time. This has to be done very precisely that both the times (calculated and measured) do not differ more than 1 minute.

K has to be calculated for every design temperature $T_{a\,spe}$ and after the iterative calculation process we end up with 8 different values of K (1 for $400^0$C, 1 for $450^0$C……1 for $750^0$C) for each specimen tested.

**How to determine the influence of temperature, coating thickness and section factor on heat transfer coefficient K**

Coefficient K is dependent on the thermal characteristics of the intumescent coating. In order to find the relationship between K and the parameters of steel temperature, coating thickness and section factor, we use Equation [12]. We now have to determine the constants $C_o$, $C_1$, $C_2$ and $C_3$ in Equation [12] in such a way that for every test specimen at every design steel temperature the best fit is found for K.

The most efficient and effective way to determine the constants $C_0$, $C_1$….$C_3$ is to apply linear regression techniques. The thermal conductivity coefficient $\lambda$ is the response variable and design temperature, coating thickness and section factor are the predictors. As $\lambda$ equals Kd (see Equation [6]), K d is the actual response variable. Therefore we have to multiply all K values, as derived by the iteration process, with the dry film thickness of the coating before starting the linear regression analysis to resolve 'K'.

After the linear regression is completed and all of the constants $C_0$….$C_3$ have been found, we are now ready to calculate K for every test specimen and for every design temperature. The result is fascinating, because the K value obtained from the measured data may differ from the K value achieved by calculation using Equation [12].

The next step is to calculate the time to achieve the design temperatures on basis of the K values achieved by calculation using Equation [12]. Depending on the quality of the fit the times to achieve a certain design temperature may differ significantly. This is not a problem if the time that is based on the K obtained by calculation is less than the time based on the measured data, ie. in this case the predicted time is less than the measured time in the fire resistance test. Hence, the result is on the safe side.

However, if the predicted time is more than the measured time in the fire resistance test, eg. if the time based on the K obtained by calculation is more than the time measured, this might be a cause for concern. Therefore the constant $C_0$ in Equation [12] has to be modified by hand in such a way that higher values for K will be obtained which means that the heat transfer to the steel section is reduced less than has been proven in the fire test.

Modification of the constant $C_o$ drags the performance of the material down over the full range of design temperatures, over the full range of Hp/A values and over the full range of coating thicknesses.

It is noted that $C_o$ is the only constant that should be modified in order to comply with the acceptance criteria.