A CONNECTION ELEMENT FOR MODELLING END-PLATE CONNECTIONS IN FIRE

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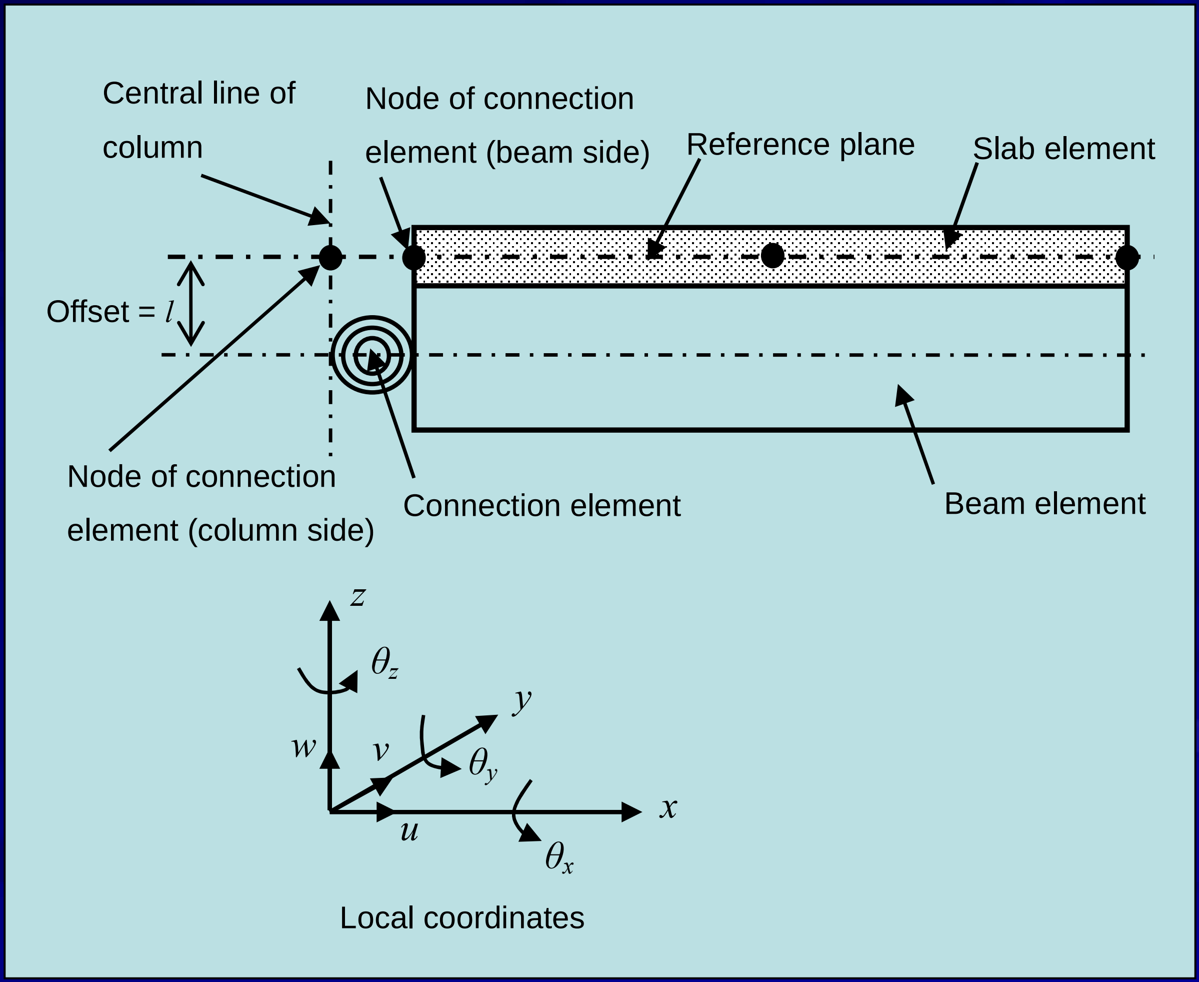
1. INTRODUCTION

Three approaches for modelling the behaviour of connections in fire

- To represent the moment–rotation characteristics of a connection by mathematical expression in the form of curve-fitting equations.
- To use component-based (also known as spring-stiffness) models for predicting the connection's behaviour.
- To model the connection as assembly of 3D finite shell, brick and contact elements using general commercial software, such as ABAQUS or ANSYS.

2. DEVELOPMENT OF THE BOLTED END-PLATE CONNECTION ELEMENT

Two-noded connection element configuration



Stiffness matrix of connection element

$$\Delta \mathbf{F} = \mathbf{K}\, \Delta \mathbf{u} \qquad (1)$$

$$\begin{Bmatrix} \Delta F_{x,1} \\ \Delta F_{y,1} \\ \Delta F_{z,1} \\ \Delta M_{x,1} \\ \Delta M_{y,1} \\ \Delta M_{z,1} \\ \Delta F_{x,2} \\ \Delta F_{y,2} \\ \Delta F_{z,2} \\ \Delta M_{x,2} \\ \Delta M_{y,2} \\ \Delta M_{z,2} \end{Bmatrix} = \begin{bmatrix} k_{11} & 0 & 0 & 0 & -k_{11}\ell & 0 & -k_{11} & 0 & 0 & 0 & k_{11}\ell & 0 \\ 0 & k_{22} & 0 & 0 & 0 & 0 & 0 & -k_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & k_{33} & 0 & 0 & 0 & 0 & 0 & -k_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & k_{44} & 0 & 0 & 0 & 0 & 0 & -k_{44} & 0 & 0 \\ -k_{11}\ell & 0 & 0 & 0 & (k_{55}+k_{11}\ell^2) & 0 & k_{11}\ell & 0 & 0 & 0 & -(k_{55}+k_{11}\ell^2) & 0 \\ 0 & 0 & 0 & 0 & 0 & k_{66} & 0 & 0 & 0 & 0 & 0 & -k_{66} \\ -k_{11} & 0 & 0 & 0 & k_{11}\ell & 0 & k_{11} & 0 & 0 & 0 & -k_{11}\ell & 0 \\ 0 & -k_{22} & 0 & 0 & 0 & 0 & 0 & k_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & -k_{33} & 0 & 0 & 0 & 0 & 0 & k_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & -k_{44} & 0 & 0 & 0 & 0 & 0 & k_{44} & 0 & 0 \\ k_{11}\ell & 0 & 0 & 0 & -(k_{55}+k_{11}\ell^2) & 0 & -k_{11}\ell & 0 & 0 & 0 & (k_{55}+k_{11}\ell^2) & 0 \\ 0 & 0 & 0 & 0 & 0 & -k_{66} & 0 & 0 & 0 & 0 & 0 & k_{66} \end{bmatrix} \begin{Bmatrix} \Delta u_1 \\ \Delta v_1 \\ \Delta w_1 \\ \Delta \theta_{x,1} \\ \Delta \theta_{y,1} \\ \Delta \theta_{z,1} \\ \Delta u_2 \\ \Delta v_2 \\ \Delta w_2 \\ \Delta \theta_{x,2} \\ \Delta \theta_{y,2} \\ \Delta \theta_{z,2} \end{Bmatrix} \qquad (2)$$

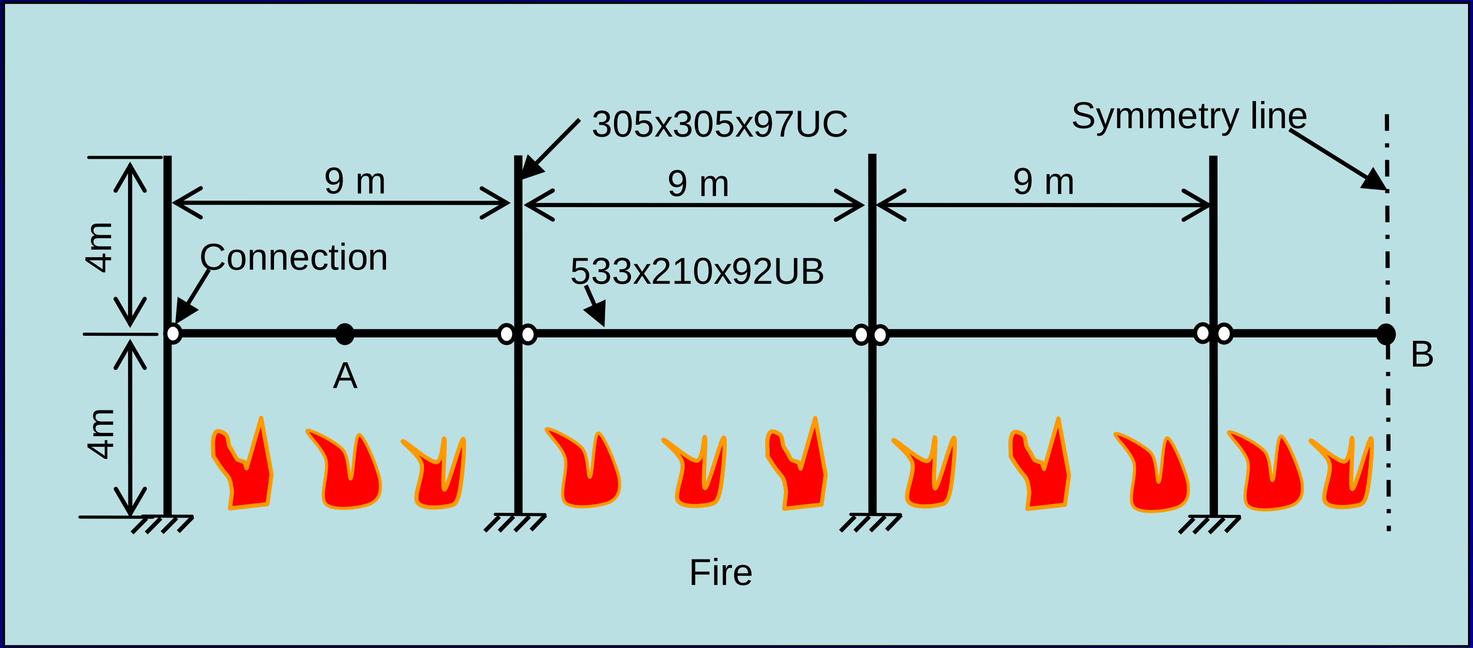
Stiffness matrix of connection element

- In this model only the in plane (*x-z* plane) behaviour of the connection is considered.

- It is reasonable to assume that the stiffness coefficients of k_{22}, k_{44}, k_{66} in Eq. (2) have infinite magnitude (assumed to be $10^9\ kN/mm$)

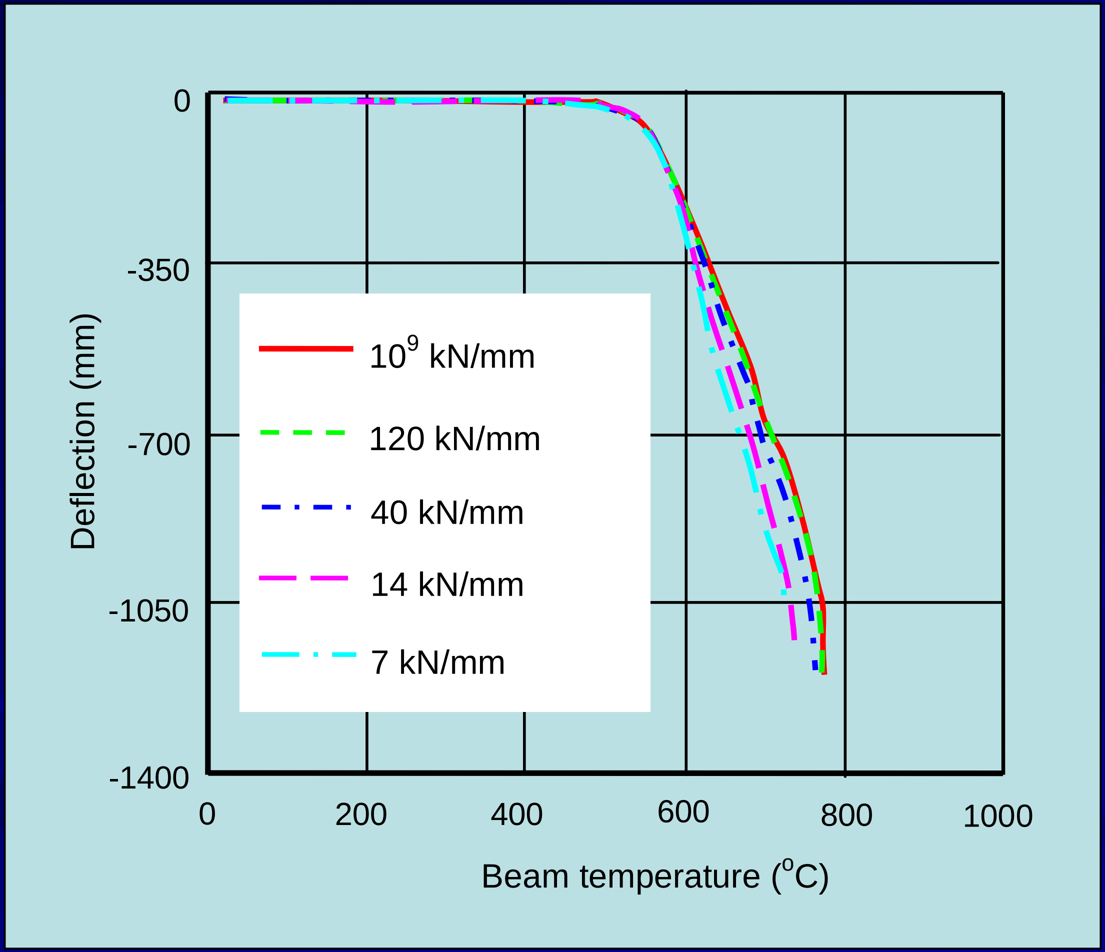
Modelled two-dimensional steel frame in fire

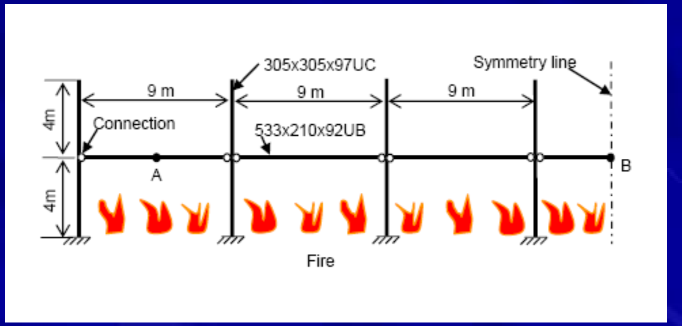
The columns were fire protected and beams were uniformly heated with load of 25 *kN/m*



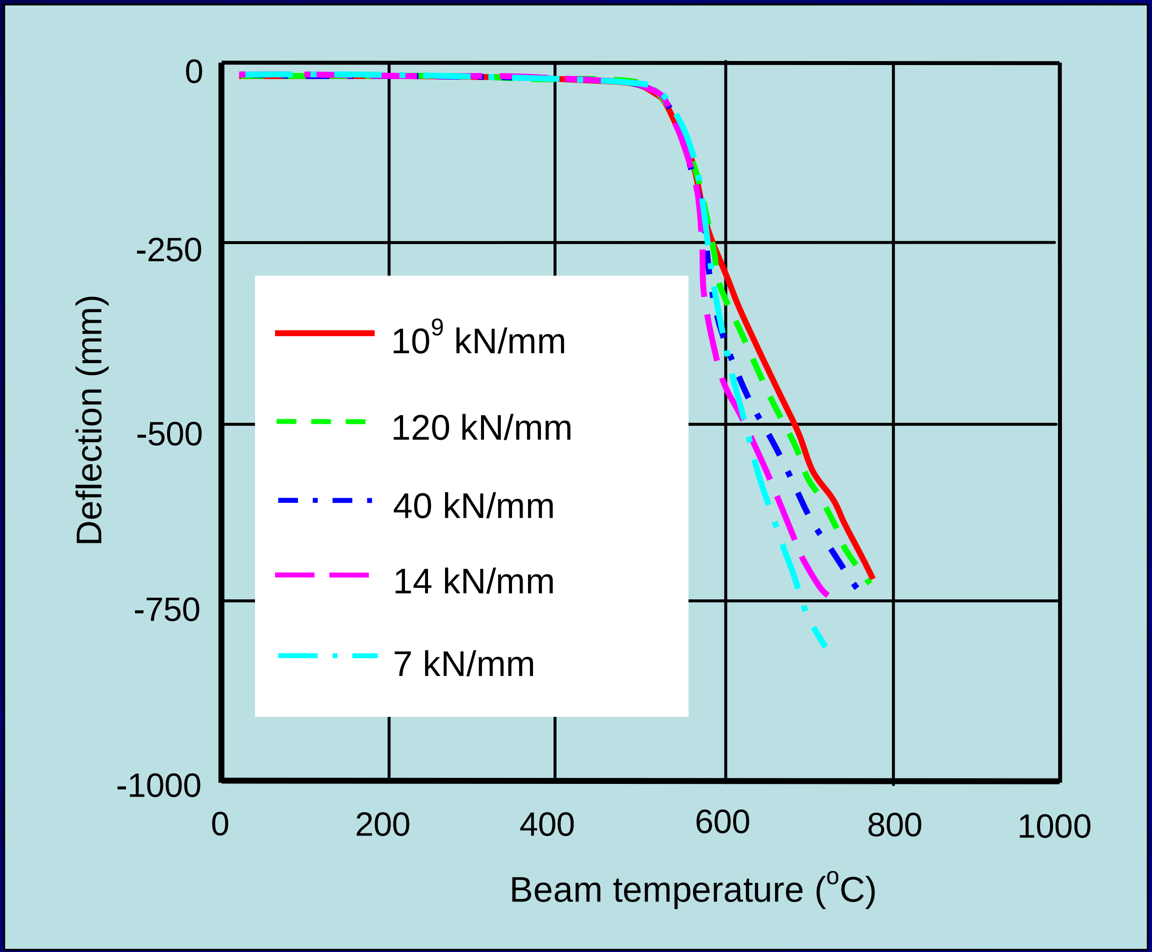
The connection was represented as an axial pinned or rigid spring with different stiffnesses for modelling pinned or rigid connection

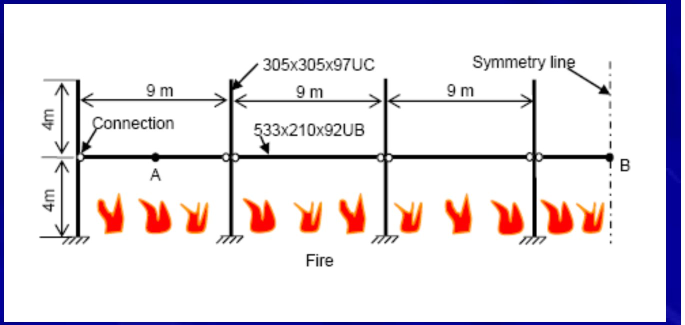
Predicted deflections at Position A for the connections using axial pinned spring with different stiffnesses



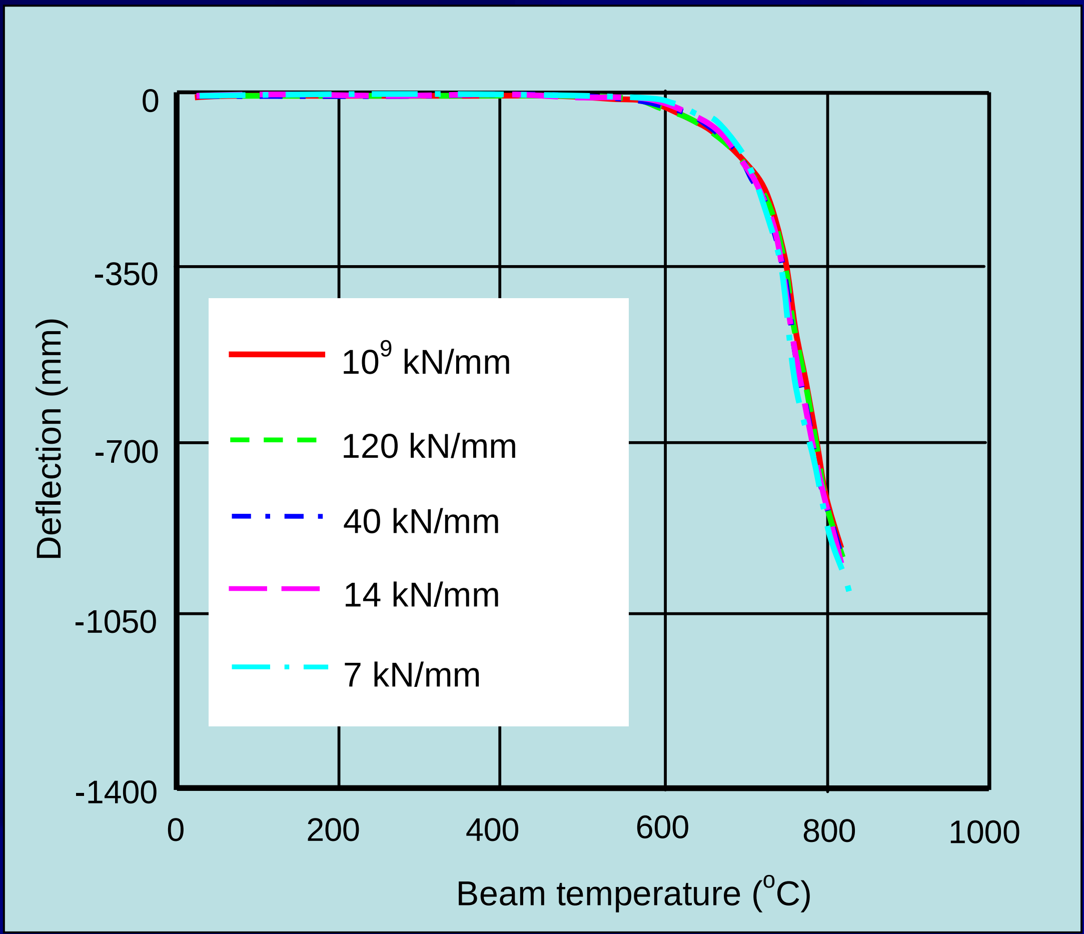


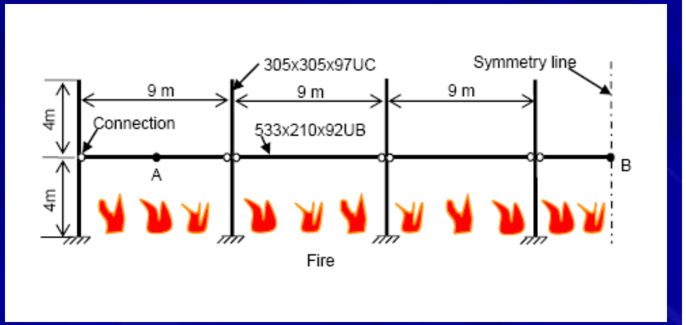
Predicted deflections at Position B for the connections using axial pinned spring with different stiffnesses



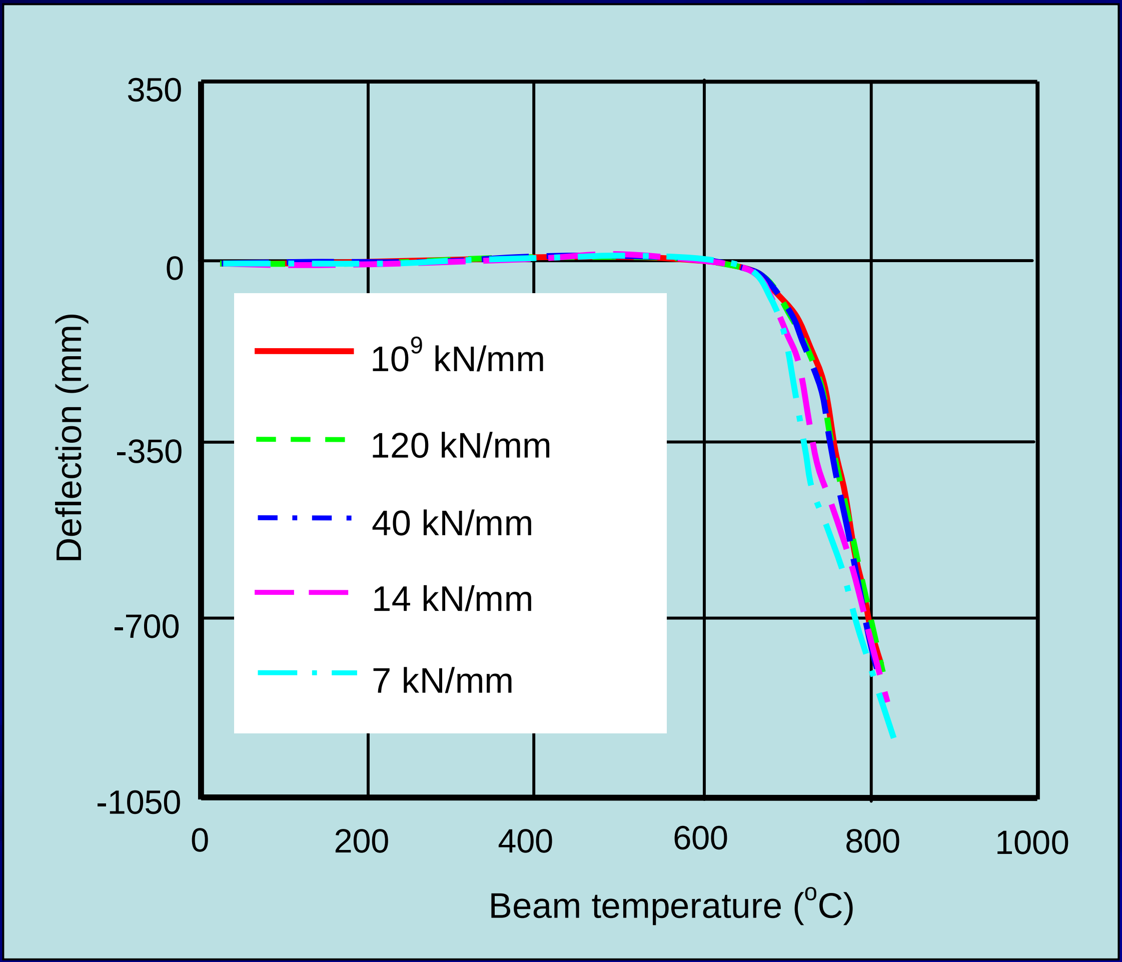


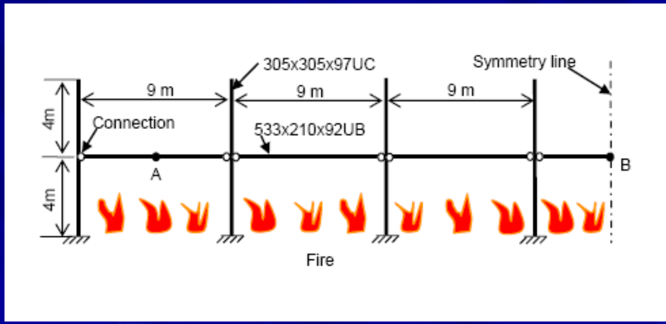
Predicted deflections at Position A for the connections using axial rigid spring with different stiffnesses





Predicted deflections at Position B for the connections using axial rigid spring with different stiffnesses

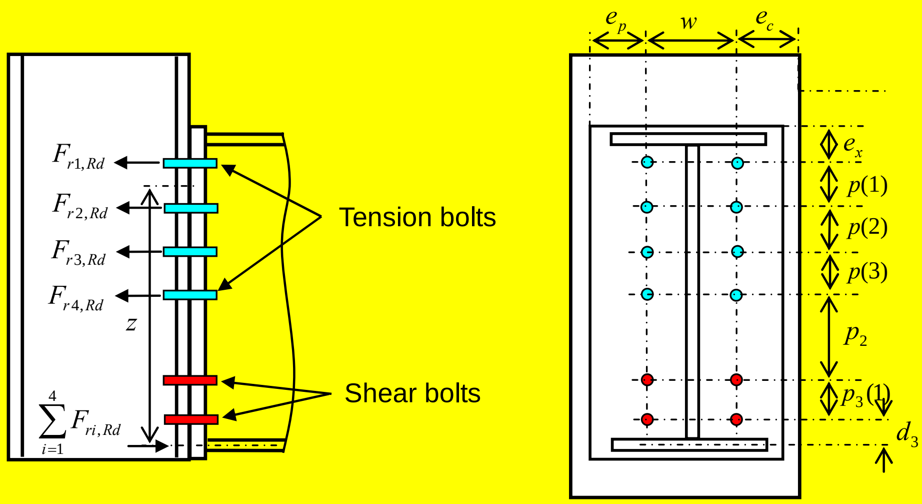




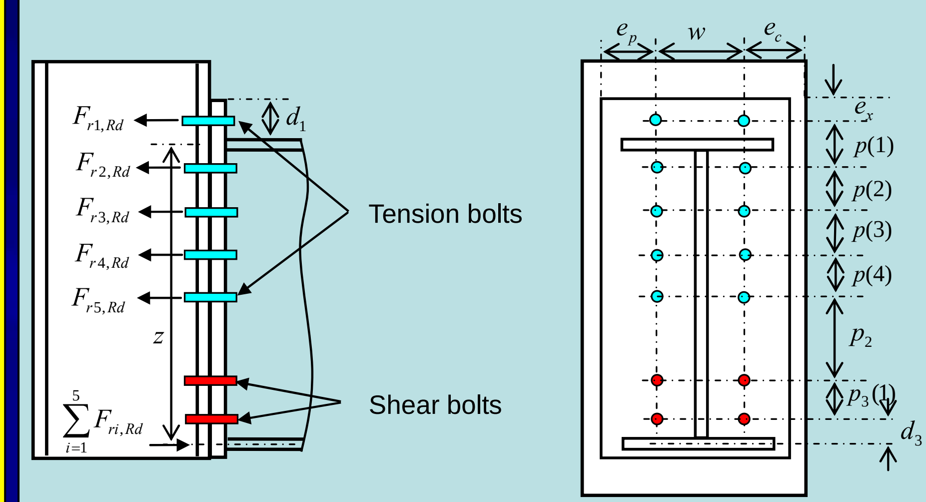
Determination of axial and vertical stiffness coefficients, k_{11} , k_{33}

- A very simplified approach was used for the current model to determine k_{11} , k_{33}:
  - Before the connection failure, $k_{11} = 10^9 \ kN/mm$
  - When the connection fails due to tension, bending or vertical shear, $k_{11} = 0$
  - When the connection failed by compression, $k_{11} = 10^9 \ kN/mm$
  - Before the connection failure due to vertical shear, $k_{33} = 10^9 \ kN/mm$
  - After the connection fails by vertical shear, $k_{33} = 0$

The detail of bolted end-plate connection between steel column and beam



(a) Flush end plate



(b) Extended end plate

Rotational stiffness of a beam-to-column connection, S_j

$$S_j = \frac{E z^2}{\mu \sum_i \frac{1}{k_i}} \qquad (3)$$

E = Young's module

k_i = stiffness coefficient for basic connection component *i*

z = lever arm

μ = stiffness ratio, $S_{j,ini} / S_j$

The initial rotational stiffness $S_{j,ini}$ of the connection is given by Eq. (3) with $\mu = 1$

$$S_{j,ini} = \frac{E z^2}{\sum_i \frac{1}{k_i}} \qquad (4)$$

Column web panel in shear (k_1)

For unstiffened single-side or double-sided connection in which the beam depths are similar:

$$k_1 = \frac{0.38\, A_{VC}}{\beta\, z} \qquad (5)$$

β = transformation parameter

A_{VC} = shear area of the column

Column web in compression (k_2)

$$k_2 = \frac{0.7\, b_{eff,c,wc}\, t_{wc}}{d_{c,c}} \qquad (6)$$

$d_{c,c}$ = clear depth of the column web

t_{wc} = thickness of the column web

$b_{eff,c,wc}$ = effective width of the column web in compression

Column web in tension (k_3)

$$k_3 = \frac{0.7\, b_{eff,t,wc}\, t_{wc}}{d_{c,c}} \qquad (7)$$

$b_{eff,t,wc}$ = effective width of the column web in tension and equal to the effective length, $\ell_{eff,c}$

$\ell_{eff,c}$ is calculated based on bolt-row considered individually for this bolt-row for an unstiffened column flange

End-plate in bending (k_5)

$$k_5 = \frac{0.9\,\ell_{eff,p}\,t_p^3}{m_p^3} \qquad (8)$$

$t_p =$ thickness of end-plate

$\ell_{eff,p}$ = effective lengths for an end-plate

m_p = a parameter related to geometry of the connection

Bolts in tension (for a single bolt-row, k_{10})

$$k_{10} = \frac{1.6\,A_s}{L_b} \qquad (9)$$

A_s = tensile stress area of the bolt

L_b = the bolt elongation length

Equivalent stiffness coefficient, k_{eq}

$$k_{eq} = \frac{\sum_r k_{eff,r} h_r}{z_{eq}} \qquad (10)$$

h_r = distance between bolt-row *r* and the centre of compression

$k_{eff,r}$ = effective stiffness coefficient of bolt-row *r*

$$k_{eff,r} = \frac{1}{\sum_i \frac{1}{k_{i,r}}} = \frac{1}{\frac{1}{k_{3,r}} + \frac{1}{k_{4,r}} + \frac{1}{k_{5,r}} + \frac{1}{k_{10,r}}} \qquad (11)$$

z_{eq} = equivalent lever arm

$$z_{eq} = \frac{\sum_r k_{eff,r} h_r^2}{\sum_r k_{eff,r} h_r} \qquad (12)$$

Rotational stiffness for one bolt-row in tension, S_j

$$S_j = \frac{E z^2}{\mu \sum_i \frac{1}{k_i}} = \frac{E z^2}{\mu \left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \frac{1}{k_4} + \frac{1}{k_5} + \frac{1}{k_{10}} \right)} \qquad (13)$$

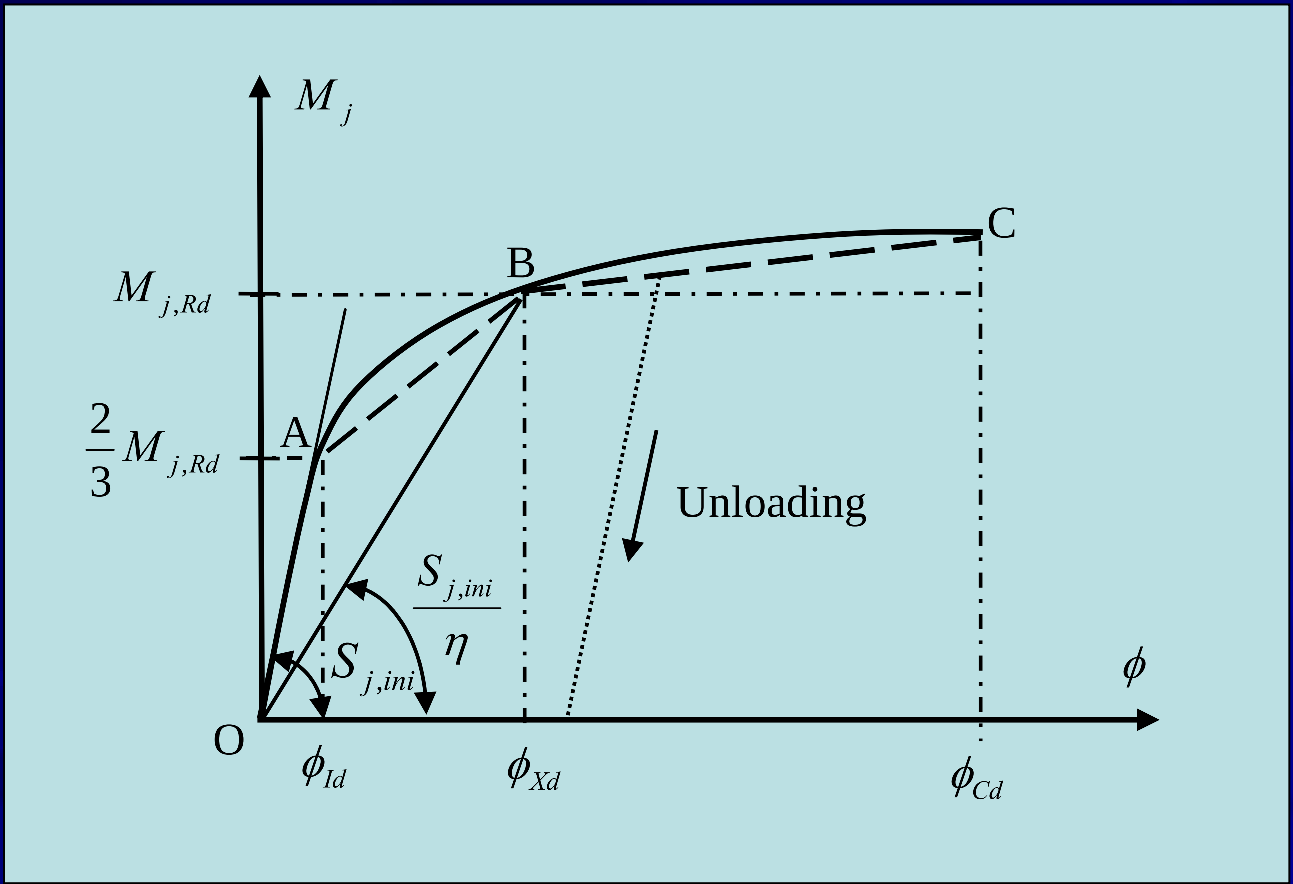
Rotational stiffness for two or more bolt-rows in tension, S_j

$$S_j = \frac{E z^2}{\mu \sum_i \frac{1}{k_i}} = \frac{E z^2}{\mu \left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_{eq}} \right)} \qquad (14)$$

E = average Young's module for the connection and changes with temperature

$$E = \frac{E_{cw} + E_{cf} + E_{bw} + E_{bf} + E_p}{5} \qquad (15)$$

Tri-linear moment-rotation characteristic used for the connection element



$$\phi_{Id} = \frac{2M_{j,Rd}}{3S_{j,ini}}$$

$$\phi_{Xd} = \frac{2M_{j,Rd}}{S_{j,ini}}$$

$$\phi_{Cd} = 5\phi_{Xd}$$

For line OA ($\phi \leq \phi_{Id}$):

$$M_j = k_{55}\,\phi = S_{j,ini}\,\phi \qquad (16)$$

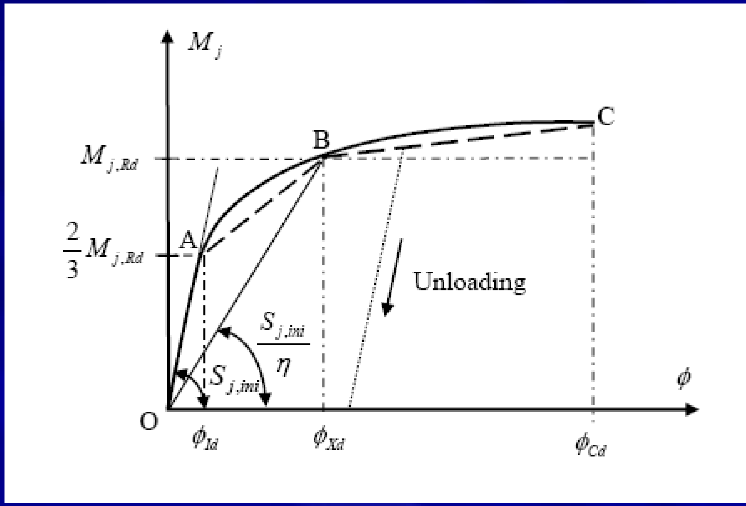
For line AB ($\phi_{Id} < \phi \leq \phi_{Xd}$):

$$M_j = k_{55}\left(\phi - \phi_{Id}\right) + \frac{2}{3} M_{j,Rd} \qquad (17)$$

$$k_{55} = \frac{M_{j,Rd}}{3\left(\phi_{Xd} - \phi_{Id}\right)}$$

For line BC ($\phi_{Xd} < \phi \leq \phi_{Cd}$):

$$M_j = k_{55}\left(\phi - \phi_{Xd}\right) + M_{j,Rd} \qquad (18)$$



$$k_{55} = 0.065\,S_{j,ini}$$

Resistance of bolt rows in the tension zone, $F_{tr,Rd}$

$$F_{tr,Rd} = \min\left(F_{t,fc,Rd}\,;\,F_{t,wc,Rd}\,;\,F_{t,ep,Rd}\,;F_{t,wb,Rd}\right) \quad (19)$$

$F_{t,fc,Rd}$ – the column flange in bending

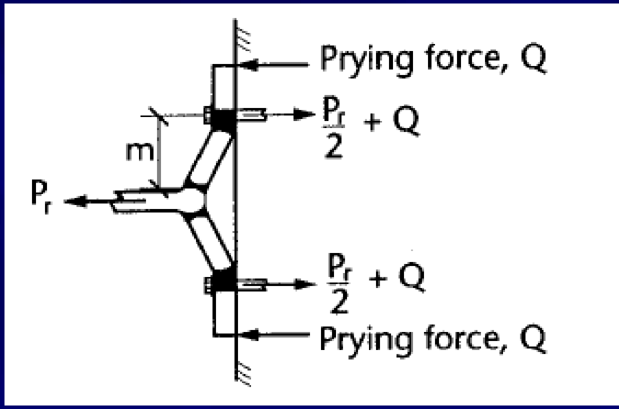
$F_{t,wc,Rd}$ – the column web in tension

$F_{t,ep,Rd}$ – the end-plate in bending

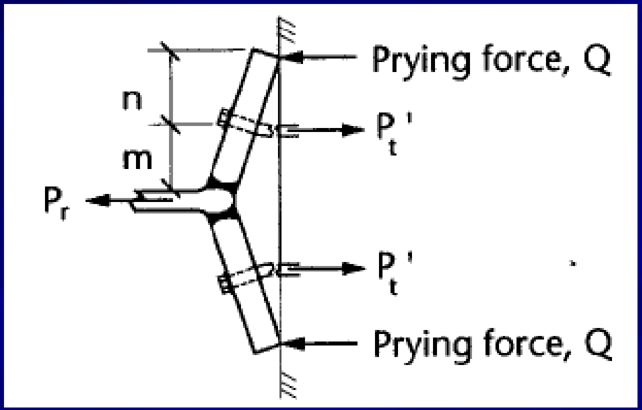
$F_{t,wb,Rd}$ – the beam web in tension

Using equivalent T-stub in tension model

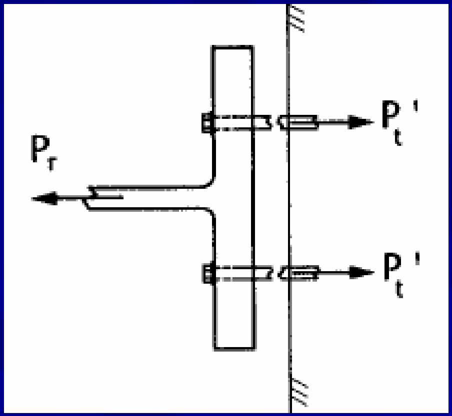
There are three failure modes:



Mode 1: complete flange yielding



Mode 2: bolt failure with flange yielding



Mode 3: bolt failure

Column flange in bending, $F_{t,fc,Rd}$

For failure Mode 1 (without backing plates):

$$M_{pl,1,Rd,r,c} = \frac{0.25 l_{eff,c} t_{fc}^{2} f_{y,c}}{\gamma_{M0}} \qquad (20)$$

$$F_{T,1,Rd,fc} = \frac{4 M_{pl,1,Rd,r,c}}{m_c} \qquad (21)$$

$t_{fc} =$ thickness of the column flange

$f_{y,c} =$ yield strength of column

For failure Mode 2:

$$M_{pl,2,Rd,r,c} = \frac{0.25 l_{eff,c} t_{fc}^{2} f_{y,c}}{\gamma_{M0}} \qquad (22)$$

$$F_{T,2,Rd,fc} = \frac{2M_{pl,2,Rd,r,c} + n_{p,c} \sum F_{t,Rd}}{m_c + n_{p,c}} \qquad (23)$$

m_c and $n_{p,c}$ are parameters related to geometry of the connection

For failure Mode 3:

$$F_{T,3,Rd,fc} = \sum F_{t,Rd} \qquad (24)$$

$F_{t,Rd}$ = resistance of individual bolt

$$F_{t,Rd} = \frac{k_2 f_{ub} A_s}{\gamma_{M2}} \qquad (25)$$

$k_2 = 0.9$

f_{ub} = ultimate tensile strength of the bolt,

A_s = tensile area of the bolt,

$$F_{t,fc,Rd} = \min\left(F_{T,1,Rd,fc};\, F_{T,2,Rd,fc};\, F_{T,3,Rd,fc}\right) \qquad (26)$$

Column web in transverse tension, $F_{t,wc,Rd}$

$$F_{t,wc,Rd} = \frac{\omega\, b_{eff,t,wc}\, t_{wc}\, f_{y,c}}{\gamma_{M0}} \qquad (27)$$

ω = reduction factor to allow for the interaction with shear in the column web panel

$$\omega = \frac{1}{\sqrt{1 + 1.3\left(\frac{b_{eff,c,wc}\, t_{wc}}{A_{vc}}\right)^2}} \qquad (28)$$

End plate in bending, $F_{t,ep,Rd}$

For failure Mode 1 (without backing plates):

$$M_{pl,1,Rd,r,b} = \frac{0.25 l_{eff,p} t_p^{\,2} f_{y,p}}{\gamma_{M0}} \qquad (29)$$

$$F_{T,1,Rd,ep} = \frac{4 M_{pl,1,Rd,r,b}}{m_{p1}} \qquad (30)$$

$f_{y,p} =$ yield strength of end-plate

For failure Mode 2:

$$M_{pl,2,Rd,r,b} = \frac{0.25 l_{eff,p} t_p^{\,2} f_{y,p}}{\gamma_{M0}} \qquad (31)$$

$$F_{T,2,Rd,ep} = \frac{2M_{pl,2,Rd,r,b} + n_{p,ep} \sum F_{t,Rd}}{m_{p1} + n_{p,ep}} \qquad (32)$$

m_{p1} and $n_{p,ep}$ are parameters related to geometry of the connection

For failure Mode 3:

$$F_{T,3,Rd,ep} = \sum F_{t,Rd} \qquad (33)$$

The resistance of the end plate in bending is

$$F_{t,ep,Rd} = \min\left(F_{T,1,Rd,ep};\ F_{T,2,Rd,ep};F_{T,3,Rd,ep}\right) \qquad (34)$$

Beam web in tension, $F_{t,wb,Rd}$

$$F_{t,wb,Rd} = \frac{b_{eff,t,wb}\, t_{wb} f_{y,b}}{\gamma_{M0}} \qquad (35)$$

$b_{eff,t,wb}$ = effective width of the beam web in tension and equal to $l_{eff,p}$

t_{wb} = thickness of the beam web

$f_{y,b}$ = yield strength of beam

Compression Resistance, $F_{c,Rd}$

$$F_{c,Rd} = \min\left(F_{c,wc,Rd}\,;\, F_{c,fb,Rd}\right) \quad (36)$$

$F_{c,wc,Rd}$ – column web in transverse compression

$F_{c,fb,Rd}$ – beam flange and web in compression

Resistance of column web in transverse compression, $F_{c,wc,Rd}$

For an unstiffened column web:

$$F_{c,wc,Rd} = \min\left(\frac{\omega k_{wc} b_{eff,c,wc} t_{wc} f_{y,c}}{\gamma_{M0}} ; \frac{\omega k_{wc} \rho b_{eff,c,wc} t_{wc} f_{y,c}}{\gamma_{M1}} \right) \qquad (37)$$

k_{wc} = reduction factor

ρ = reduction factor for plate buckling

Resistance of Beam flange and web in compression, $F_{c,fb,Rd}$

$$F_{c,fb,Rd} = \frac{M_{c,Rd}}{\left(h_b - t_{fb}\right)} \qquad (38)$$

h_b = depth of the connected beam

$M_{c,Rd}$ = moment resistance of the beam cross-section

t_{fb} = flange thickness of the connected beam

If the height of the beam including the haunch exceeds 600 mm the contribution of the beam web to the compression resistance should be limited to 20%:

$$F_{c,fb,\max} \leq \frac{b_b\, t_{fb}\, f_{y,b}}{0.8\gamma_{M0}} \qquad (39)$$

b_b = width of the beam section

$$F_{c,fb,Rd} \leq F_{c,fb,\max} \qquad (40)$$

Force distribution in bolt rows

The first condition that the effective tension resistance has to satisfy is:

$$F_{c,Ed} \leq F_{c,Rd} \qquad (41)$$

$$F_{c,Ed} = \sum_{r=1}^{N} F_{tr,Rd} \qquad (42)$$

N = total number of bolt rows in tension

If $F_{c,Ed} > F_{c,Rd}$ the force distribution in bolt rows should be adopted to make sure that:

$$F_{c,Ed} = \sum_{r=1}^{N} F_{tr,Rd} = F_{c,Rd} \qquad (43)$$

Moment resistance, $M_{j,Rd}$

$$M_{j,Rd} = \sum_r h_r F_{tr,Rd} \qquad (44)$$

$F_{tr,Rd}$ = effective design tension resistance of bolt-row *r*

h_r = distance from bolt-row *r* to the centre of compression

r = the bolt-row number

Resistance for individual bolt subjected to vertical shear forces, $F_{b,Rd}$

$$V_{b,Rd} = \min\left(F_{v,Rd};\ F_{b,cf,Rd};\ F_{b,ep,Rd}\right) \qquad (45)$$

$F_{v,Rd}$ – shear resistance of one bolt

$F_{b,cf,Rd}$ – bolts in bearing on column flange

$F_{b,ep,Rd}$ – bolts in bearing on end plate

$$F_{v,Rd} = \frac{\alpha_V f_{u,bolt} A_s}{\gamma_{M2}} \qquad (46)$$

$f_{u,bolt} =$ ultimate tensile strength of bolt

For classes 4.6, 5.6 and 8.8 bolts: $\alpha_V = 0.6$

For classes 4.8, 5.8, 6.8 and 10.9 bolts: $\alpha_V = 0.5$

$$F_{b,cf,Rd} = \frac{k_{1,c}\alpha_{b,c} f_{u,c} d\, t_{fc}}{\gamma_{M2}} \quad (47)$$

$$F_{b,ep,Rd} = \frac{k_{1,p}\alpha_{b,p} f_{u,p} d\, t_{p}}{\gamma_{M2}} \quad (48)$$

$d =$ nominal bolt diameter

$f_{u,b} =$ ultimate tensile strength of column

$f_{u,p} =$ ultimate tensile strength of end-plate

Resistance of the connection with the axial force $N_{j,Ed}$ in the connected beam

$$\frac{M_{j,Ed}}{M_{j,Rd}} + \frac{N_{j,Ed}}{N_{j,Rd}} \leq 1.0 \qquad (49)$$

$M_{j,Rd}$ = moment resistance of the connection, no axial force

$N_{j,Rd}$ = axial resistance of the connection, no applied moment

The moment resistance of the connection which consider the influence of axial force:

$$M'_{j,Rd} = \left(1.0 - \frac{N_{j,Ed}}{N_{j,Rd}}\right) M_{j,Rd} \qquad (50)$$

The axial resistance of the connection which consider the influence of applied moment:

$$N'_{j,Rd} = \left(1.0 - \frac{M_{j,Ed}}{M_{j,Rd}}\right) N_{j,Rd} \qquad (51)$$

For tension resistance:

$$N_{j,Rd} = \sum_{r=1}^{N} F_{tr,Rd} \qquad (52)$$

For compression resistance:

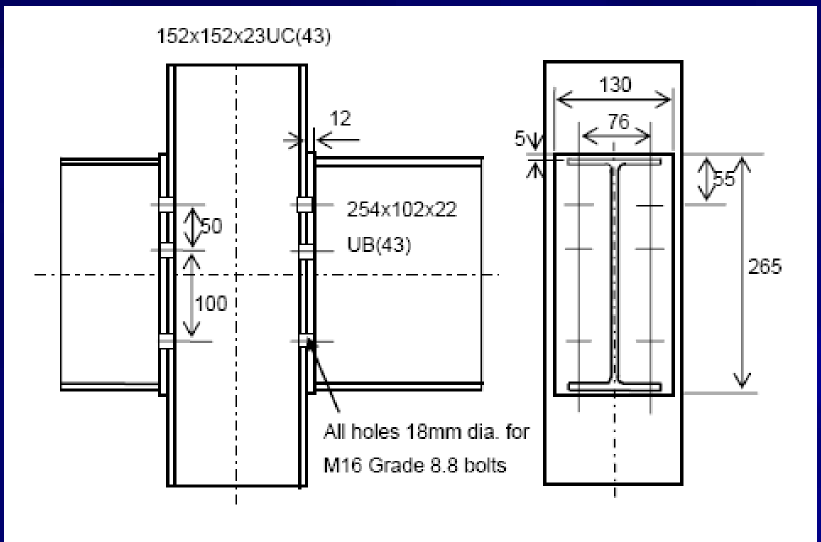
$$N_{j,Rd} = F_{c,Rd} \qquad (53)$$

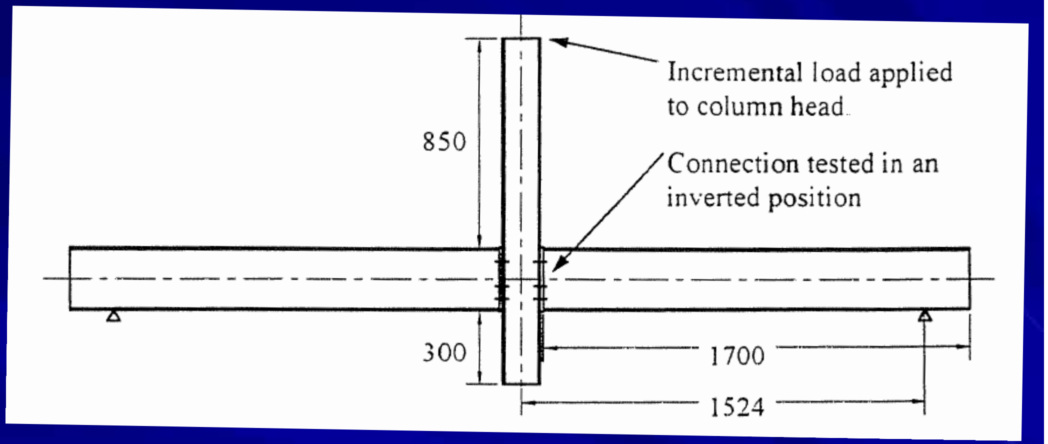
Connection behaviours at elevated temperatures

- The model presented above can be extended into elevated temperatures by relating all material properties, such as yield strength; ultimate tensile strength and Young’s module to the temperature.
- It is assumed that the material degradation of bolt at elevated temperatures is the same for the structural steel.
- The model specified in Eurocode 3 Part 1.2 is adopted in this research.

3. VALIDATIONS

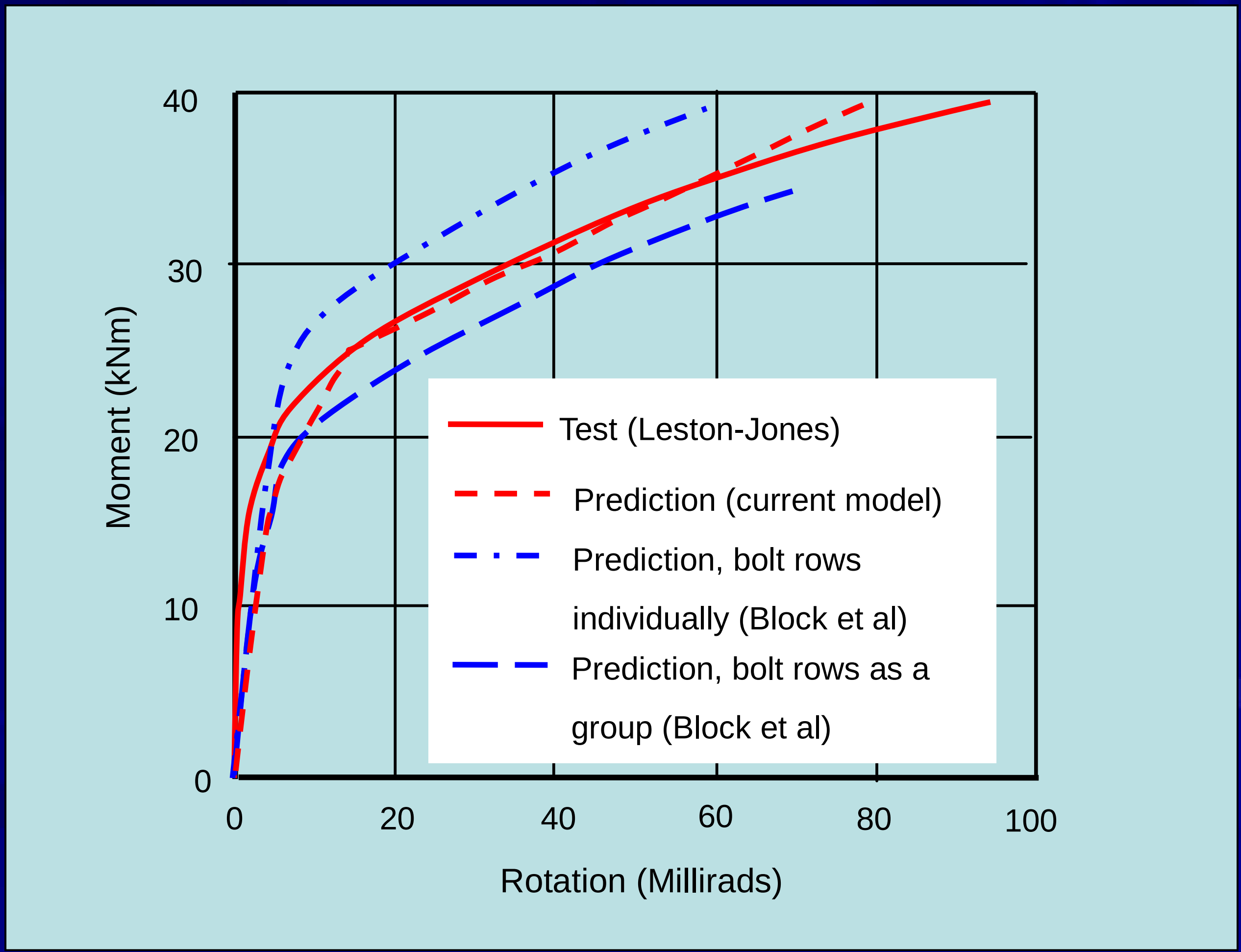
Bolted end-plate connection tested at ambient temperature



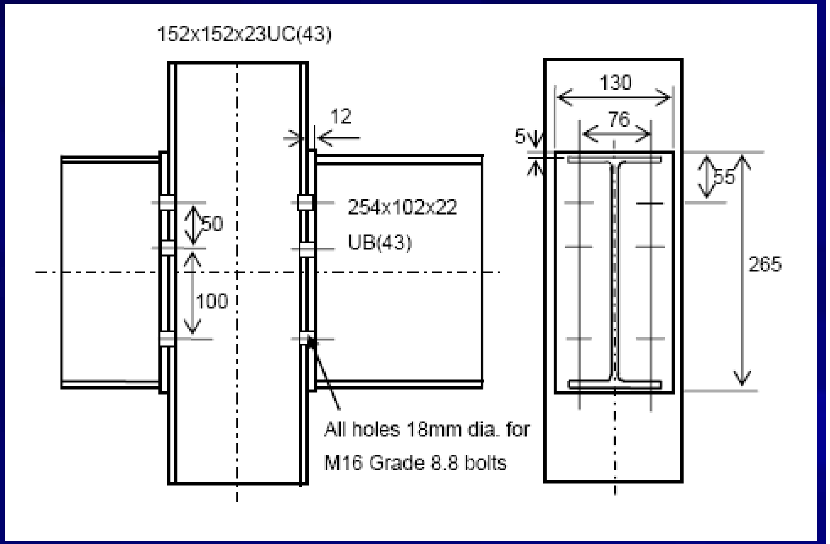


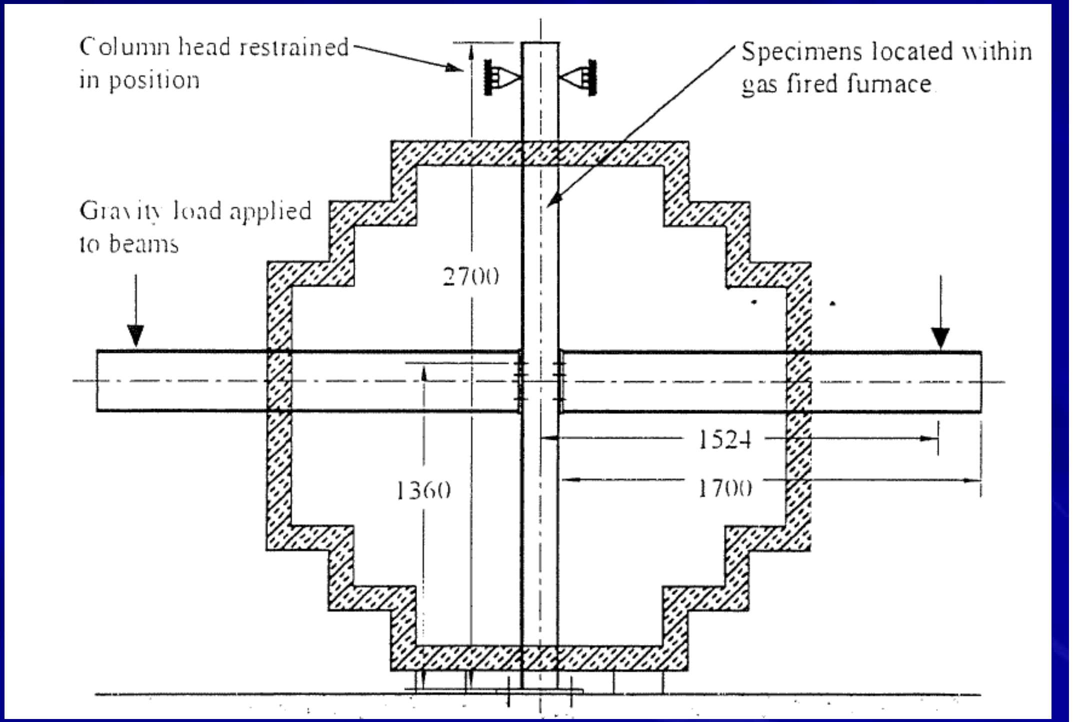
The detail of ambient temperature test (Leston-Jones 1997)

Comparison of predicted and measured moment-rotation curves (Leston-Jones 1997)



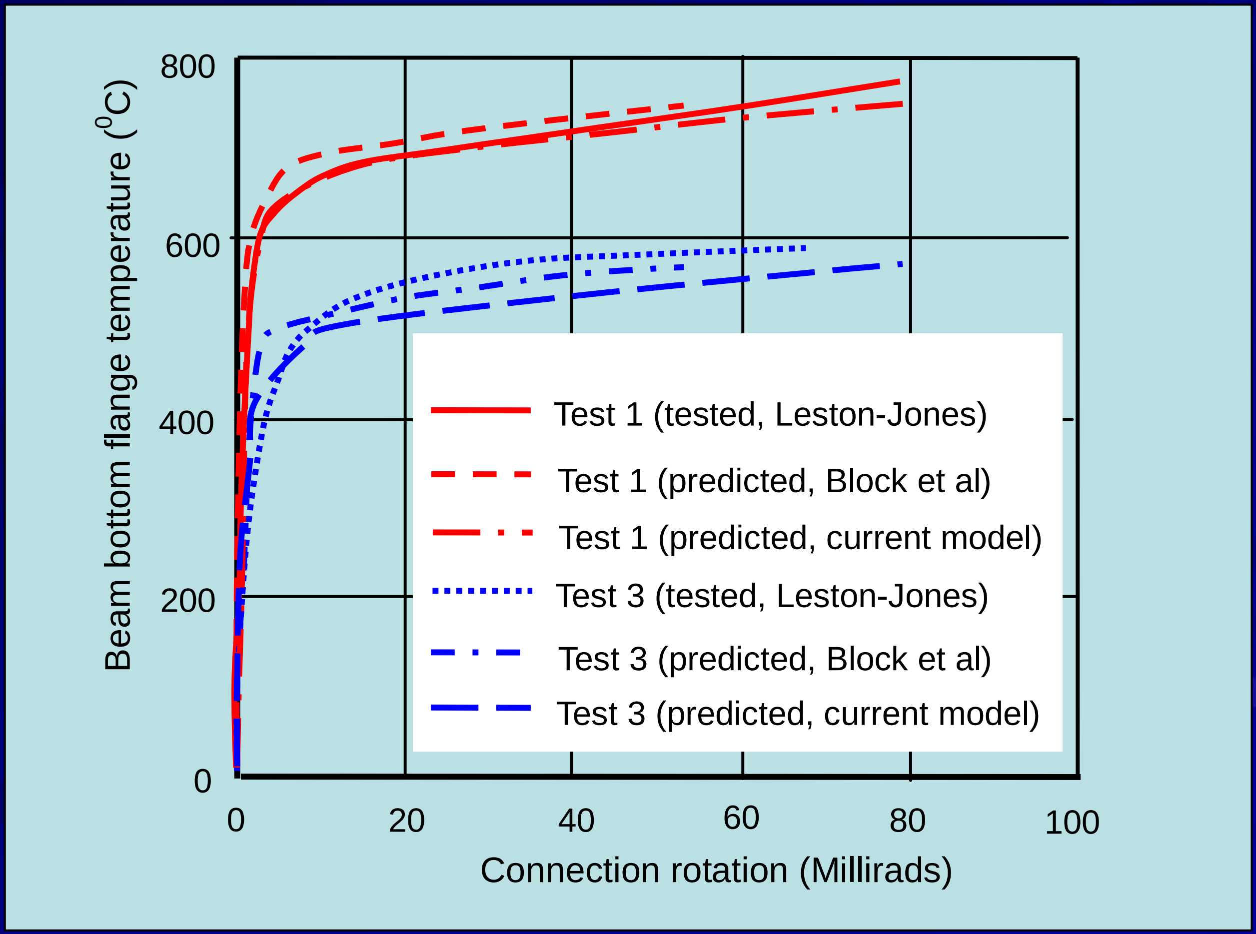
The detail of tests at elevated temperatures (Leston-Jones 1997)



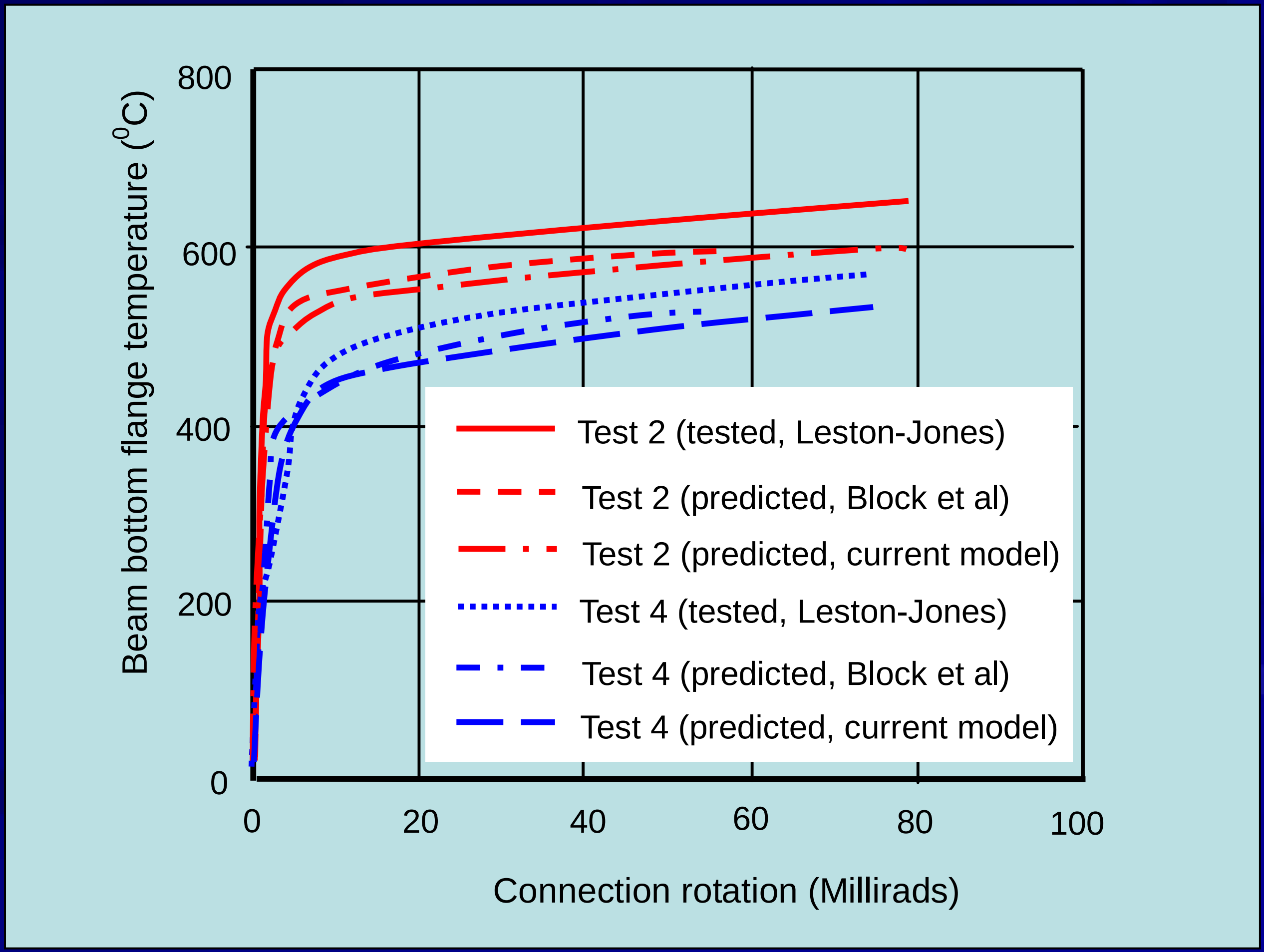


The load levels applied to Test 1, Test 2, Test 3 and Test 4 were 5 kNm, 10 kNm, 15 kNm and 20 kNm, respectively

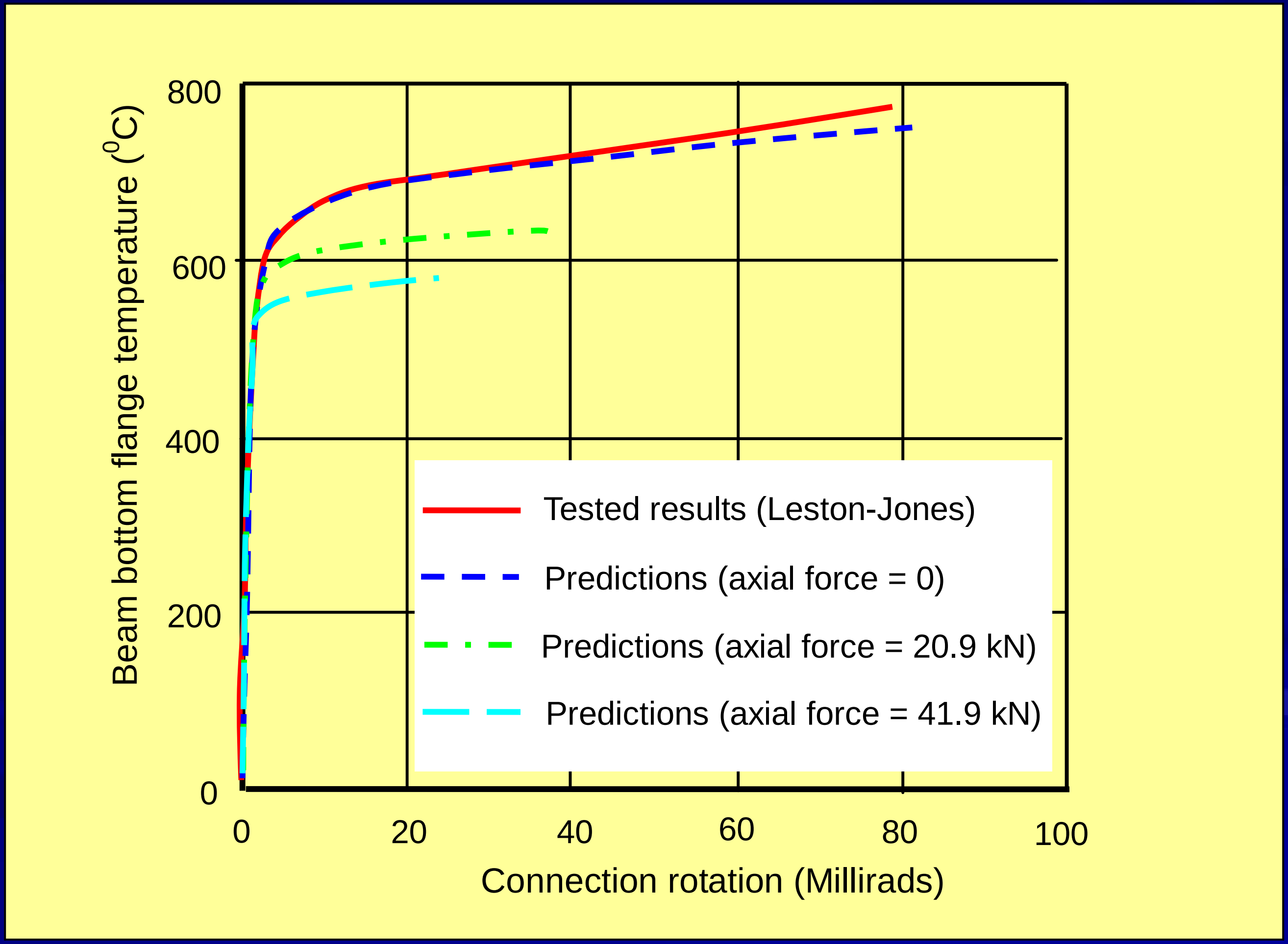
Comparison of predicted and measured connection rotations for Test 1 and Test 3 (Leston-Jones 1997)



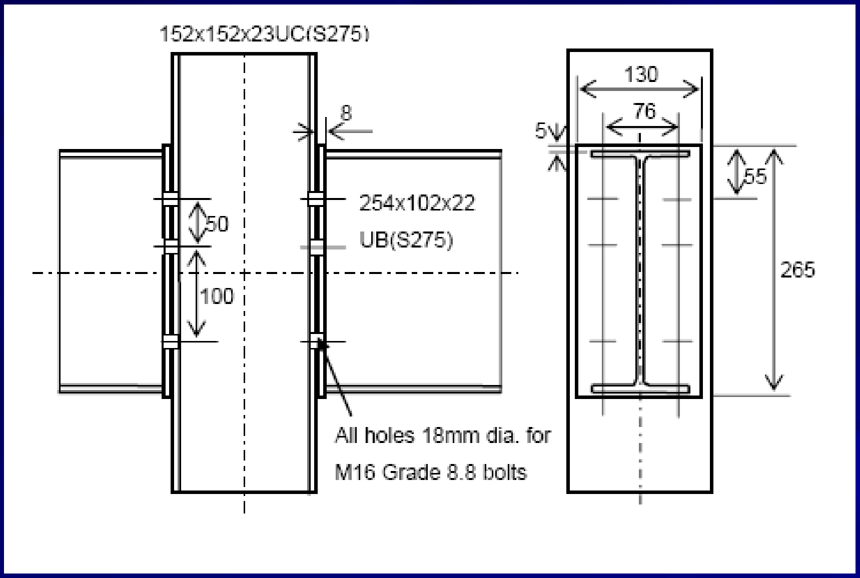
Comparison of predicted and measured connection rotations for Test 2 and Test 4 (Leston-Jones 1997)

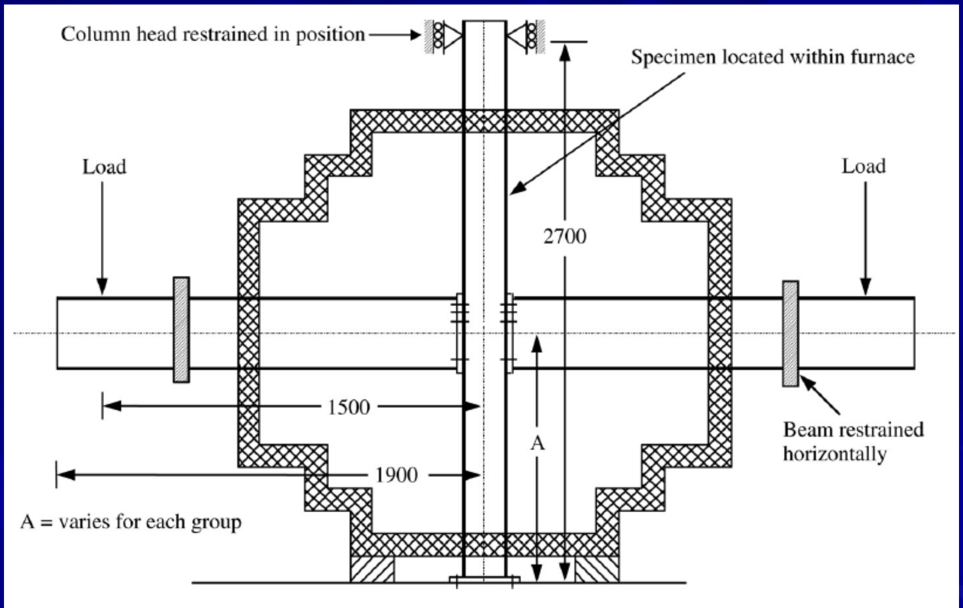


Influence of beam axial forces on the connection behaviour for Test 1 (Leston-Jones 1997)



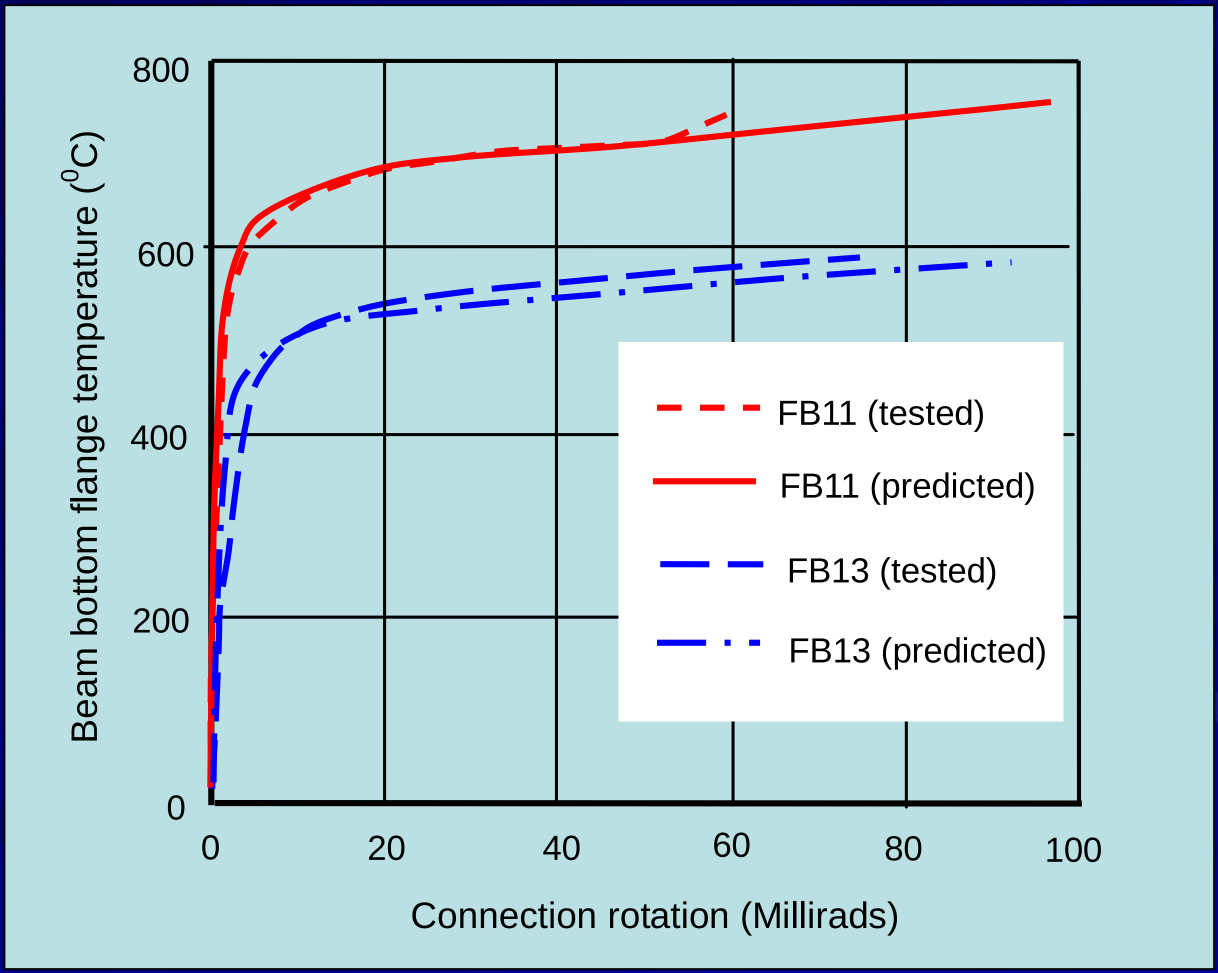
The detail of Group 1 (FB1) connection fire tests (Al-Jabri et al 2005)



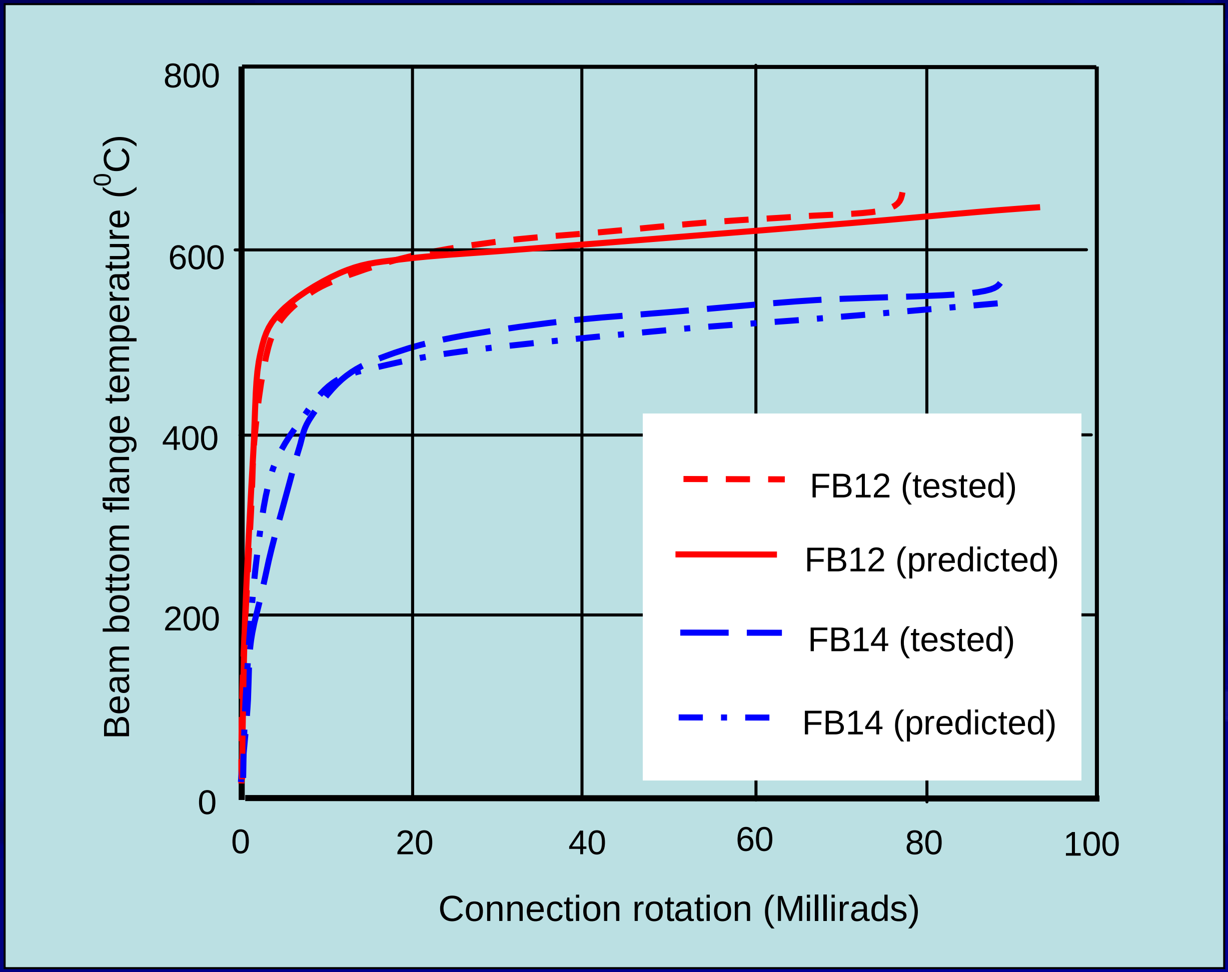


Four tests FB11, FB12, FB13, FB14 were conducted at load levels corresponding to connection moments of 4.4 kNm, 8.2 kNm, 13.12 kNm and 17.1 kNm, respectively

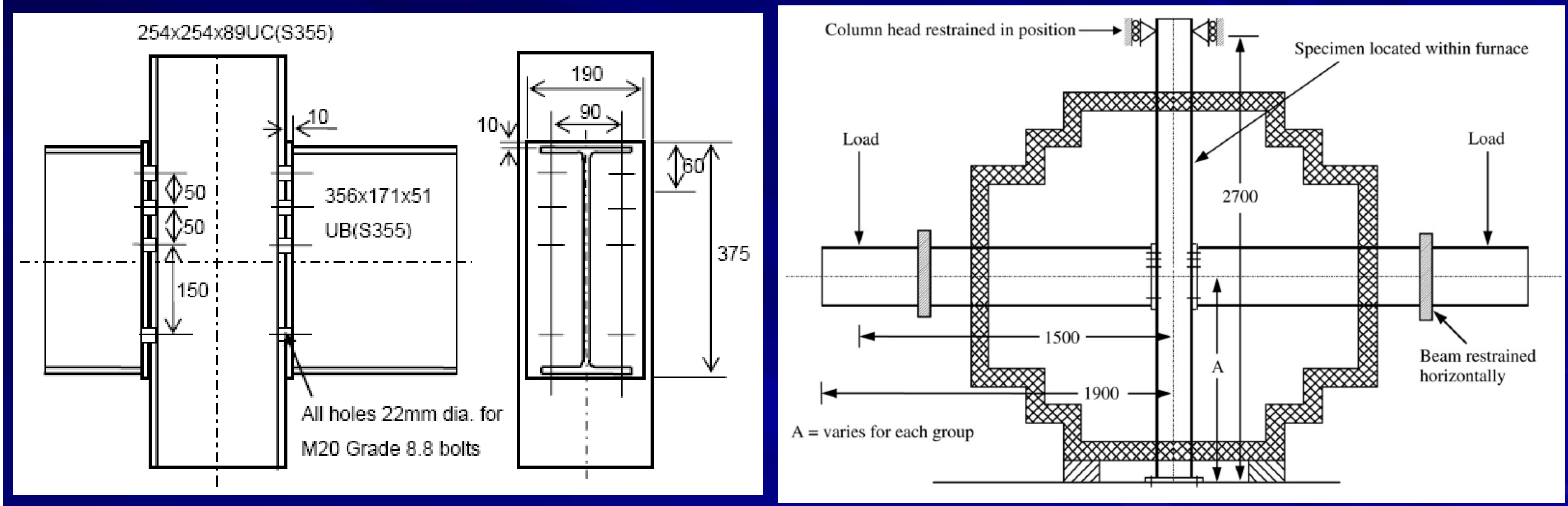
Comparison of predicted and measured connection rotations for Group 1: FB11, FB13 (Al-Jabri et al 2005)



Comparison of predicted and measured connection rotations for Group 1: FB12, FB14 (Al-Jabri et al 2005)

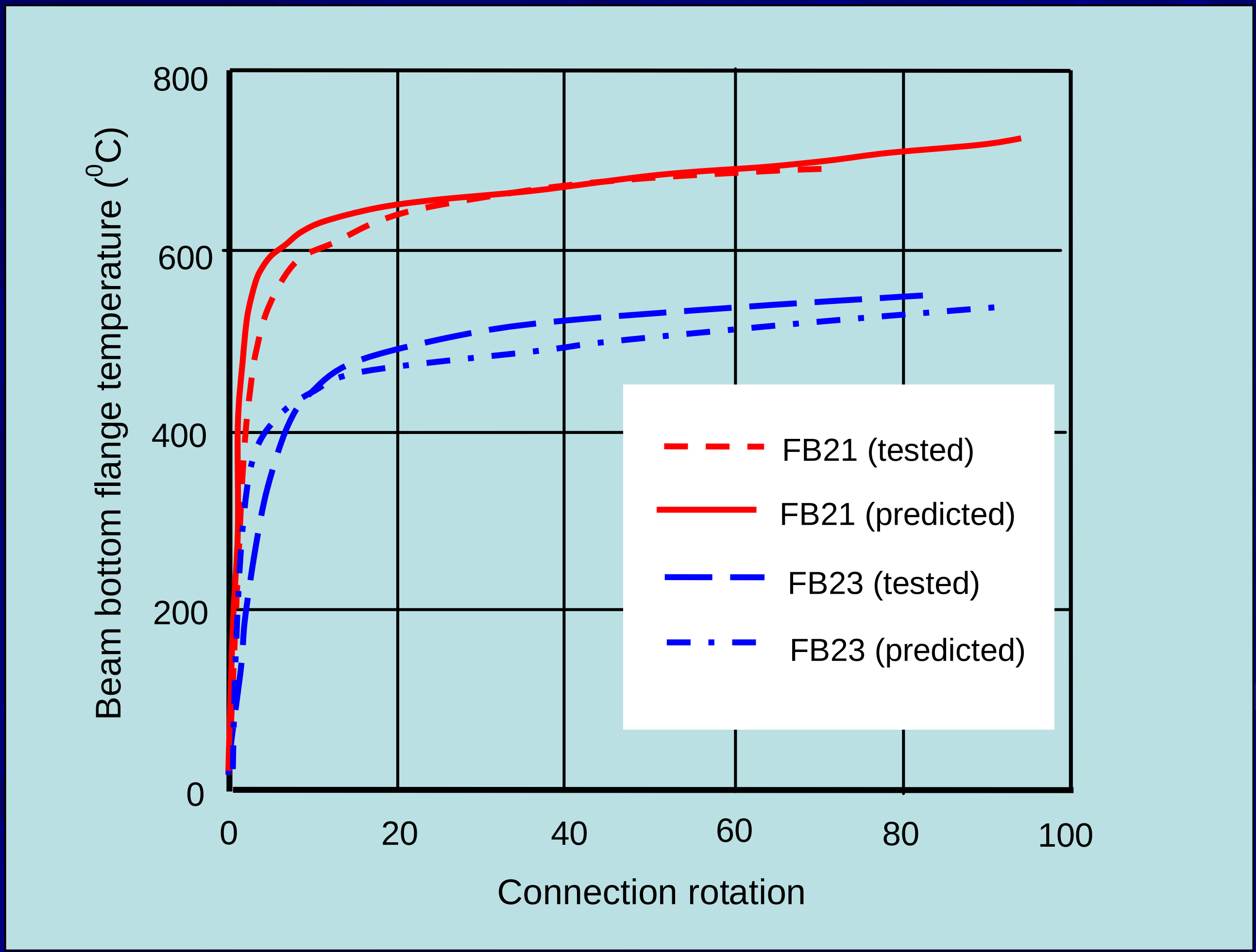


The detail of Group 2 (FB2) connection fire tests (Al-Jabri et al 2005)

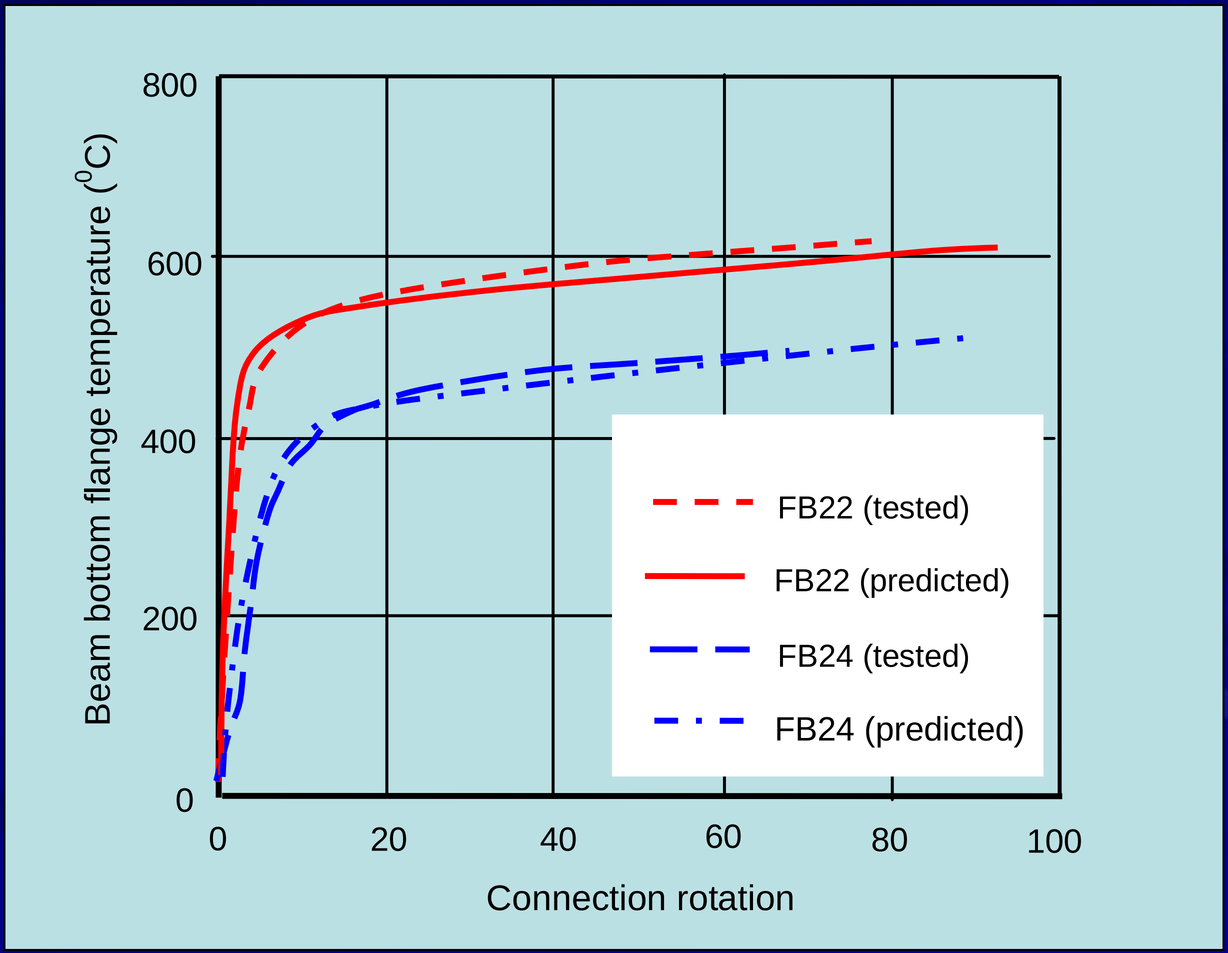


Four tests denoted as FB21, FB22, FB23, FB24 were carried out using the load levels of 27.4 kNm, 54.8 kNm, 82.1 kNm and 110 kNm, respectively

Comparison of predicted and measured connection rotations for Group 2: FB21, FB23 (Al-Jabri et al 2005)



Comparison of predicted and measured connection rotations for Group 2: FB22, FB24 (Al-Jabri et al 2005)



4. CONCLUSIONS

- The current model has the advantages of both the previous simple and component-based models.
- The current model is robust and has a capability to predict the behaviour of bolted end-plate connection under fire attack with reasonable accuracy.
- Compared to the tested results the predictions of the current model were mainly on conservative side.
- The model can be used for structural fire engineering design on steel-framed composite buildings.
- The idea described in this paper can also easily be applied to develop other kind of connections.

Thank You